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The role of expectations in economic fluctuations and the efficacy of monetary policy

Mordecai Kurz[a,*], Hehui Jin[a], Maurizio Motolese[b]

[a]Department of Economics, Stanford University, Stanford, CA 94305-6072, USA
[b]Istituto di Politica Economica, Universitá Cattolica di Milano, Via Necchi 5, 20123, Milano, Italy

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Abstract

Diverse beliefs is an important mechanism for propagation of fluctuations, money nonneutrality and efficacy of monetary policy. Since expectations affect demand, our theory shows economic fluctuations are mostly driven by varying demand not supply shocks. Using a competitive model with flexible prices in which agents hold Rational Belief (see Kurz, 1994. Economic Theory 4, 877–900) we show that (i) our economy replicates well the empirical record of fluctuations in the U.S. (ii) Under monetary rules without discretion, monetary policy has a strong stabilization effect and an aggressive anti-inflationary policy can reduce inflation volatility to zero. (iii) The statistical Phillips curve changes substantially with policy instruments and activist policy rules render it vertical. (iv) Although prices are flexible, money shocks result in less than proportional change in inflation hence aggregate price level is 'sticky' with respect to money shocks. (v) Discretion in monetary policy adds a random element to policy and increases volatility. The impact of discretion on the efficacy of policy depends upon the structure of market beliefs about future discretionary decisions. We study two rationalizable beliefs. In one, market beliefs weaken the effect of policy and in the second, beliefs bolster policy outcomes and discretion could be a desirable attribute of the policy rule. Since the central bank does not know any more than the private sector, discretion is beneficial only in extraordinary cases. Hence, the weight of the argument suggests that policy *should be*

*Corresponding author. Tel.: +1 650 723 2220; fax: +1 650 7255702.
E-mail address: mordecai@stanford.edu (M. Kurz).

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transparent and abandon discretion except for rare circumstances. (vi) Our model suggests the current real policy is only mildly activist and aims mostly to target inflation.

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1. Introduction

What explains the observed real effect of money on the economy and is money not neutral? This is perhaps the most debated question of our time. Empirical evidence has demonstrated that monetary policy, unanticipated and anticipated (e.g. Mishkin, 1982), has real effects and virtually all countries established economic stabilization as the main goal of central bank policy. However, if we seek a scientific justification for this policy, we find sharp differences in models, assumptions and methods used to arrive at this conclusion.

On one side is the standard rational expectations (in short, RE) based real business cycle theory which holds that all real fluctuations are caused by exogenous real technological shocks, money is neutral and only relative prices matter for economic allocation. Under this theory, anticipated monetary policy cannot have real effect and hence stabilizing monetary policy cannot provide any long term and consistent social benefits (e.g. see Lucas, 1972; Sargent and Wallace, 1975).

An opposing view holds that money is not neutral, that economic fluctuations impose a policy tradeoff between inflation and unemployment and such a 'Phillips curve' is at the foundation of economic stabilization policy. This perspective has been developed by the *Dynamic New Keynesian* (*in short DNK*) *Theory* which erected the Keynesian view on three pillars: (1) the market consists of price setting monopolistically competitive firms, (2) prices are 'sticky' due to restrictions on firms' ability to adjust prices (e.g. Taylor, 1980, 1993, 1999; Calvo, 1983; Yun, 1996; Goodfriend and King, 1997; Bernanke et al., 1999; Clarida et al., 1999; Levin et al., 1999; Mankiw and Reis, 2002; McCallum and Nelson, 1999; Rotemberg and Woodford, 1999; Woodford, 2001, 2003a), and (3) markets are complete, agents are identical and hold RE within a Rational Expectations Equilibrium (in short, REE). Most work with Calvo's (1983) idealization where at any date only a fraction of firms are 'allowed' to change prices while others cannot. In such an economy output fluctuations are caused by exogenous shocks and amplified by incorrect firms' price setting. This monopolistic competitive equilibrium is not Pareto efficient. Changes in nominal rates have real effects because they impact *expected* future prices by firms. An exogenous shock causes some firms to change prices but others cannot adjust them and must produce output given prices set earlier, based on expectations held at that date and are thus the 'wrong' prices today. Monetary policy aims to restore efficiency by countering the negative effect of price rigidity. Depending upon the model of price stickiness, this objective implies that central bank aims to set nominal rates at each date so the resulting equilibrium private sector expected inflation equals the rate anticipated by agents forced to fix prices in the previous date.

We share the DNK theory's view that monetary policy is a very useful stabilization tool. However, this paper shows an important cause for the efficacy of monetary policy is the heterogeneity of market expectations rather than price inflexibility or monopolistic competition in price setting. An argument in support of the efficacy of monetary policy would consist of three parts:

(A) In a market economy agents make socially undesirable allocation decisions resulting in excess fluctuations of inflation and real variables hence a component of business cycle fluctuations is man made, endogenously propagated by the actions of market participants.

(B) Money is not neutral: changes in the nominal rate impact aggregate excess demand.

(C) Monetary policy can help stabilize the endogenous component of fluctuations.

In what economies do conclusions (A)–(C) hold? Under the assumptions of (i) frictionless perfect competition, (ii) flexible prices and (iii) REE allocations, conclusions (A)–(C) cannot be reached: money is neutral and monetary policy has no social function. To deduce (A)–(C), some of these assumptions must be modified. The DNK theory rejects the first two, postulating instead a monopolistic price setting and price inflexibility. We preserve the assumptions of perfect competition and price flexibility hence our model economy is standard. However, we remove the homogeneous belief assumption and deduce our results from the assumption that agents hold heterogenous beliefs about state variables. In fact, even if a monetary policy rule is transparent and there are no differences of opinion about what the rule is, agents make different price forecasts since they forecast different values of the state variables. Our equilibrium is a Radner equilibrium (Radner, 1972) with an expanded state space, a development explained in detail in this paper. We restrict beliefs by requiring them to satisfy the rationality principle of rational belief (in short RB or RBE for 'rational belief equilibrium') developed by Kurz (1994) and others in Kurz (1996, 1997a). Since heterogeneity of beliefs is the driving force of our theory, we provide here a short review of the RB perspective.

1.1. The Rational Belief principle

'Rational Belief' *is not a theory which demonstrates rational agents should adopt any specific belief*. In fact, since the RB theory explains the observed heterogeneity of beliefs, it would be a contradiction to propose that any particular belief is the 'correct' belief which rational agents must adopt. The RB theory starts by observing that the true stochastic law of motion of the economy is a nonstationary process with structural breaks and complex dynamics and the probability law of this process is not known by anyone. Agents have a long history of data generated by the process in the past which they use to compute relative frequencies of finite dimensional events and correlation among observed variables. With this knowledge they compute the empirical distribution of observed variables and use it to construct an empirical probability measure over sequences. Since all these measures are based on the law of large numbers, it is a theorem that this estimated probability model must be stationary. In the RB theory it is called the 'empirical measure' or the 'stationary measure.'

In contrast with REE where the true law of motion is known, agents in an RBE form beliefs based only on available data. Hence, any principle on the basis of which agents can be judged as rational must be based *on the data rather than on the true but unknown law of motion*. Since a 'belief' is a model of the economy together with a probability measure over sequences of variables, such a model can be simulated to generate artificial data. With simulated data the agent can compute the empirical distribution of observed variables and hence the empirical probability measure the model implies. Based on these facts, the RB theory proposes a simple Principle of Rationality. It says that if the agent's model does not reproduce the empirical distribution known for the real economy, then the agent's model (i.e. 'belief') is declared irrational. The contra-positive is also required to hold: for a belief to be rational its simulated data must reproduce the known empirical distribution of the observed variables. To 'reproduce' an empirical distribution means *to match all its moments*. The RB rationality means a belief is viewed as rational if it is a model which cannot be disproved with the empirical evidence. Since diverse theories are compatible with the same evidence, this rationality principle permits diversity of subjective beliefs among equally informed rational agents. Agents who hold rational beliefs may make 'incorrect' forecasts at any date but must be correct, on average. Also, date t forecasts may deviate from the forecast implied by the empirical distribution. However, since the RB rationality principle requires the long term average of an agent's forecasts to agree with the forecast based on the empirical frequencies, it follows *as a theorem* that agents who hold rational beliefs which are different from the empirical distribution must have forecast functions which vary over time. The key tool we use to describe the distribution of beliefs in an economy is the 'market state of belief' which uniquely defines the conditional probabilities of agents. Since this is a central idea of our paper, we dedicate Section 3.1 to explain it in detail.[1] We also note that RB rationality is compatible with several known theories. An REE is a special case and so are the associated REE with sunspots. Also, several models of Bayesian Learning and Behavioral Economics are special cases of an RBE and satisfy the RB rationality principle for some parameter choices.

[1]Earlier papers which used the RBE perspective have argued that most volatility in financial markets is caused by the beliefs of agents (e.g. Kurz, 1996, 1997a; Kurz and Schneider, 1996; Kurz and Beltratti, 1997; Kurz and Motolese, 2001; Kurz et al., 2005 and Nielsen, 1996). These papers introduced a unified model which explains, simultaneously, a list of financial phenomena regarded as 'anomalies' centered around the Equity Premium Puzzle. The model's key feature is the heterogeneity of agent's beliefs where the distribution of market beliefs (i.e. market 'state of belief') fluctuates over time. Phenomena such as the Equity Premium Puzzle are then explained by the fact that pessimistic 'bears' who aim to avoid capital losses drive interest rates low and the equity premium high (for a unified treatment see Kurz et al. (2005)). The RBE theory was used by Kurz (1997b) and Nielsen (2003) to explain the volatility of foreign exchange markets and by Wu and Guo (2003) to study speculation and trading volume in asset markets.

A short explanation of how an RBE leads to implications (A)–(C) above may be helpful. In a typical RBE endogenous variables depend upon the state of belief which exhibit fluctuations over time. Such fluctuations induce fluctuations of endogenous variables making them more volatile than explained by exogenous shocks. Market fluctuations are further amplified by correlation among beliefs of agents. Belief heterogeneity takes two forms: (i) diverse interpretation of information, and (ii) diverse forecasts of endogenous variables due to *diverse individual forecasts of future state of belief of others*. 'Optimistic,' agents increase the level of economic activity above normal and 'pessimistic' agents cut back on consumption, investment and production plans below normal levels. Hence, fluctuations in the market state of belief is an important market externality.

Implication (B) showing money is not neutral in an RBE is not new. It was reported by Motolese (2001, 2003) and Kurz et al. (2003) who study monetary policy in a model of random growth of money. This paper builds on Kurz et al. (2003) who demonstrate that, in an otherwise frictionless economy, diversity of beliefs can reproduce the empirical regularities observed in monetary economies. To see why money is not neutral in an RBE one refers to Lucas (1972). In this seminal contribution he showed money neutrality is fundamentally an expectational problem. To exhibit money neutrality Lucas (1972) shows one must assume common information with common beliefs across agents, all expecting money to be neutral. This property does not hold under diverse beliefs (also, see Woodford, 2003b). Hence, if common belief in neutrality of money does not hold, money is not neutral.

As for implication (C), central bank policy cannot affect fluctuations due to technology. Since money is not neutral, the excess endogenous volatility of a market economy suggests the bank can stabilize the endogenous component of fluctuations by countering the effect of private beliefs.

Rigidities and imperfections such as inflexible wages, costly input adjustments or asymmetric information certainly play some role in the efficacy of monetary policy. Such factors complement our theory: adding any of these rigidities to our theory only strengthen our conclusions. Diversity of beliefs is a propagation mechanism which generates *demand driven* real and financial market volatility. It provides a unifying paradigm to explain the propagation of business fluctuations, to clarify why monetary policy is effective and to justify the use of such policy as a stabilization tool.

This paper explores how a central bank can attain stabilization *by countering the effect of private expectations*. We examine diverse monetary policy rules in order to study their stabilization effect in our economy. The structure of this paper is as follows. In Sections 2–3 we develop a simple model (extending Kurz et al., 2005), explain the structure of beliefs and the RB restrictions. In Section 4 we study the volatility of RBE with money shocks using computational methods. We compare its volatility with the level of fluctuations of the traditional Real Business Cycles (RBC in short) model and with the economy in which money grows at a constant rate. In Section 5 we study the performance of the economy under simple Taylor (1993) type rules with and without discretion and inertia. Section 6 offers an interpretation of the efficacy of monetary policy under heterogenous beliefs and its relation to the violation of iterated expectations of market belief.

2. The economic environment

The economy has four traded goods: a consumption good, one period nominal bill, labor services and fiat money. Agents trade all goods on competitive markets. There are two types of agents and a large number of identical agents within each type. Each agent is a member of one of two types of infinitely lived dynasties identified by their labor, by their utility (which is defined over consumption, labor services and real money holding) and by their beliefs. A member of a dynasty lives a fixed short life and during his life makes decisions based on his own state of belief without knowing the states of belief of his predecessors. An agent also manages a constant returns to scale firm owned by the dynasty. The firm employs the capital stock the dynasty owns and produces consumer goods while operating in competitive markets for labor services and for short term loans. Since each firm is owned by a dynasty, the intertemporal decisions of the firm are made based on the stochastic discount factors of its owner. The income of agents consists of labor income and the income from four assets owned. First, capital owned by the agent and employed by the dynasty's firm. Second are ownership share in the dynasty's firm. These ownership shares do not trade on the open market. Third is a one period, zero net supply *nominal* bill which pays a riskless return hence it is risky with respect to the rate of inflation. Fourth is fiat money issued by a central bank.

In a monetary environment of *random money growth* each agent receives a proportional share of the money growth and we assume the mean growth rate of money equals the mean growth rate of GNP. Under a *nominal interest rate policy rule* a change in the money supply results from an endogenous change in the demand for money. In that model the target rate of inflation is set equal to 1% per quarter. We assume a government balanced budget so that all changes in money supply are financed by lump sum, per capita taxes or subsidies.

At each date, firms hire labor in competitive markets, make investment decisions and select optimal rates of capacity utilization of the capital they employ. In making investments agents can produce new capital goods by using their own savings or by borrowing on the open market to finance these projects. Investments are irreversible: once produced, capital goods cannot be turned back into consumption goods but they depreciate with use. Firms' decisions maximize discounted present value of future cash flow from producing consumer goods, given the nominal interest rate, the nominal wage rate and the prices of consumer goods. Markets for consumption good, labor and short term bonds (or bills) are competitive and all prices are flexible: no prices are sticky.

Our model is then traditional. There are no informational asymmetries. The main feature of our theory is that agents hold diverse belief, not Rational Expectations.

Since an agent owns his firm, we could bypass the firm problem by writing a grand household problem. A separate treatment is simpler and contributes to the clarity of the exposition hence we discuss the two separately.

2.1. The household problem

Our model has two infinitely lived dynasties of agents enumerated $j = 1, 2$ but for simplicity we shall refer to each one of them as 'agents j' and introduce the following notation:

C_t^j	consumption of j at t
P_t	price level or, the nominal price of a unit of the consumption good at t
ℓ_t^j	leisure of agent j at t
L_t^j	labor employed by firm j at t
$\tilde{W}_t$	nominal wage at t; $W_t = \tilde{W}_t/P_t$ – the real wage at t
ξ_t	the mean level of technological productivity at t
K_t^j	number of units of capital owned by j and employed by the firm owned by j at date t
Y_t^j	real output by firm j of consumer goods at t
I_t^j	new investments of j at date t
$P_t/P_{t-1} = e^{\pi_t}$	where π_t is the rate of inflation at t
B_t^j	amount of one period nominal bill purchased by agent j at t
r_t	the one period nominal interest rate
$q_t^b = 1/1 + r_t$	the price of a one period bill at t, which is a discount price
M_t^j	amount of money held by agent j at t
φ_t^j	rate of capacity utilization of firm j
H_t	history of all observables up to t.

Each household owns a firm with a production function which takes the form

$$Y_t^j = e^{\upsilon_t}(\varphi_t^j K_t^j)^{\sigma}(\xi_t L_t^j)^{1-\sigma}. \tag{1}$$

The productivity process $\{\xi_t, t = 1, 2 \ldots\}$ is a *deterministic* trend process satisfying

$$\frac{\xi_{t+1}}{\xi_t} = \upsilon^*. \tag{2}$$

The random productivity $\{\upsilon_{t+1}, t = 1, 2, \ldots\}$ will be specified when we study the firm's optimization. A firm carries out the household's investment. It maximizes the present value of cash flow and pays the household an amount $P_t\tilde{f}_t^j = P_t Y_t^j - \tilde{W}_t L_t^j - P_t I_t^j$, which the household considers exogenous. $P_t\tilde{f}_t^j$ is not 'dividend' as it incorporates the household's capital account *and may be negative*.

With exogenous money growth $M_{t+1}/M_t \equiv \upsilon^* e^{\varrho_{t+1}}$. Since ϱ_{t+1} has a zero mean, the long term mean inflation rate in a money growth model is zero. A similar condition applies to each agent:

$$M_t^j = M_{t-1}^j \upsilon^* e^{\varrho_t} \quad \text{for } j = 1, 2. \tag{3}$$

Under an interest rate policy money is endogenous. If j increases money holdings from M_{t-1}^j to M_t^j he pays the government $M_t^j/P_t - M_{t-1}^j/P_t$ units of consumption goods. To maintain balanced budget we introduce lump sum transfers $T_t\xi_t$ in units of consumption goods and make the 'Ricardian' assumption that $P_tT_t\xi_t$ equals the value of the newly issued money. Transfers per agent are equal.

To ensure the existence of a steady state for the economy we assume the rate of discount and degree of risk aversion are the same for all agents. If Q^j is a probability belief of j then he solves

$$\underset{(C_t^j, \ell_t^j, B_t^j, M_t^j)}{\text{Max}} \; \mathrm{E}_{Q^j}\left[\sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{1-\gamma}\left((C_t^j(\ell_t^j)^{\zeta})^{1-\gamma} + \left(\frac{M_t^j}{P_t}\right)^{1-\gamma}\right) \middle| H_t\right], \quad 0<\beta<1. \tag{4a}$$

subject to two possible budget constraints. Under an exogenous growth of the money the budgets are

$$P_tC_t^j = (1-\ell_t^j)\tilde{W}_t + P_t\tilde{f}_t^j + B_{t-1}^j + M_{t-1}^j\upsilon^* e^{\varrho_t} - B_t^j q_t^b - M_t^j, \quad j = 1, 2. \tag{4b}$$

Under an interest rate rule the budget constraint are

$$P_tC_t^j = (1-\ell_t^j)\tilde{W}_t + P_t\tilde{f}_t^j + B_{t-1}^j + M_{t-1}^j + \tfrac{1}{2}P_t(T_t\xi_t) - B_t^j q_t^b - M_t^j, \quad j = 1, 2. \tag{4c}$$

Normalizing, define

$$c_t^j = \frac{C_t^j}{\xi_t}, \quad b_t^j = \frac{B_t^j}{P_t\xi_t}, \quad w_t = \frac{\tilde{W}_t}{P_t\xi_t}, \quad i_t^j = \frac{I_t^j}{\xi_t}, \quad f_t^j = \frac{\tilde{f}_t^j}{\xi_t},$$

$$\frac{M_t^j}{P_t\xi_t} = m_t^j, \quad \frac{M_{t-1}^j}{P_t\xi_t} = \frac{m_{t-1}^j}{e^{\pi_t}\upsilon^*}$$

hence the inflation rate π_t is defined by $P_t/P_{t-1} = e^{\pi_t}$. Using (4a)–(4c) the maximization problem is

$$\underset{(c_t^j, \ell_t^j, b_t^j, m_t^j)}{\text{Max}} \; \mathrm{E}_{Q^j} \sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{1-\gamma}[(c_t^j\xi_t(\ell_t^j)^{\zeta})^{1-\gamma} + (m_t^j\xi_t)^{1-\gamma}|H_t], \quad 0<\beta<1 \tag{5a$'$}$$

subject to two possible budget constraints. Under a money growth regime the budget constraints are

$$c_t^j = (1-\ell_t^j)w_t + f_t^j + e^{(\varrho_t - \pi_t)}m_{t-1}^j + \frac{b_{t-1}^j e^{-\pi_t}}{\upsilon^*} - b_t^j q_t^b - m_t^j, \quad j = 1, 2 \tag{5b$'$}$$

and under a nominal interest rate rule regime they are

$$c_t^j = (1-\ell_t^j)w_t + f_t^j + \frac{m_{t-1}^j + b_{t-1}^j}{\upsilon^* e^{\pi_t}} - b_t^j q_t^b - m_t^j + \frac{1}{2}T_t, \quad j = 1, 2. \tag{5c$'$}$$

The first order conditions are entirely standard. For labor supply the conditions are (for $j = 1, 2$)

$$c_t^j = \frac{1}{\zeta}\,\ell_t^j w_t. \tag{6a}$$

The first order condition with respect to bond purchases b_t^j is

$$q_t^b = \mathrm{E}_{Q^j}\left[\beta(\upsilon^{*-\gamma})\left(\frac{c_{t+1}^j}{c_t^j}\right)^{-\gamma}\left(\frac{\ell_{t+1}^j}{\ell_t^j}\right)^{\zeta(1-\gamma)} e^{-\pi_{t+1}}\right]. \tag{6b}$$

The optimum with respect to money holdings *under a regime of monetary growth* requires

$$1 - \left(\frac{m_t^j}{c_t^j}\right)^{-\gamma}\frac{1}{(\ell_t^j)^{\zeta(1-\gamma)}} = \mathrm{E}_{Q^j}\left[\beta(\upsilon^{*-\gamma})\left(\frac{c_{t+1}^j}{c_t^j}\right)^{-\gamma}\left(\frac{\ell_{t+1}^j}{\ell_t^j}\right)^{\zeta(1-\gamma)}\upsilon^* e^{(\varrho_{t+1}-\pi_{t+1})}\right] \tag{6c}$$

and *under a regime of a monetary rule* it requires

$$1 - \left(\frac{m_t^j}{c_t^j}\right)^{-\gamma}\frac{1}{(\ell_t^j)^{\zeta(1-\gamma)}} = \mathrm{E}_{Q^j}\left[\beta(\upsilon^{*-\gamma})\left(\frac{c_{t+1}^j}{c_t^j}\right)^{-\gamma}\left(\frac{\ell_{t+1}^j}{\ell_t^j}\right)^{\zeta(1-\gamma)} e^{-\pi_{t+1}}\right]. \tag{6d}$$

Observe that firm j evaluates future cash flow with the stochastic discount rate of agent j defined by

$$\mu_{t+n,t}^j = \left[(\beta\upsilon^{*-\gamma})^n\left(\frac{c_{t+n}^j}{c_t^j}\right)^{-\gamma}\left(\frac{\ell_{t+n}^j}{\ell_t^j}\right)^{\zeta(1-\gamma)}\right]. \tag{7}$$

In all simulations we set $\beta = 0.99$ which is appropriate to a quarterly model; $\gamma = 2.00$ is a realistic measure of risk aversion and $\zeta = 3.00$ is the leisure elasticity. $\zeta = 3.00$ ensures the fraction of time worked in steady states is around 0.225. It implies an elasticity of labor supply (a so-called 'λ-constant elasticity') of around 1.3 which is close to the empirical estimates of this elasticity.

2.2. Technology, production and investments

We first discuss key features of the production function as defined in (1). $\{\upsilon_t, t = 1, 2, \ldots\}$ is a stochastic process under a true probability *which is nonstationary with structural breaks and time dependent distribution*. This probability is not known by any agent and is not specified. Instead, we assume $\{\upsilon_t, t = 1, 2, \ldots\}$ is a stable process[2] hence it has an *empirical distribution* which is known to all agents who learn it from the data. This empirical distribution is represented by a stationary Markov process, with a quarter as a unit of time, defined by[3]

$$\upsilon_{t+1} = \lambda_\upsilon \upsilon_t + \rho^\upsilon_{t+1}, \quad \rho^\upsilon_t \sim \mathrm{N}(0, \sigma^2_\upsilon) \text{ i.i.d.} \tag{2'}$$

Productivity is the same across all firms. Most studies estimate the quarterly mean rate of technical change at $\upsilon^* = 1.0045$ and this is the value we use. The key parameters for the traditional RBC literature are $(\sigma, \lambda_\upsilon, \sigma_\upsilon)$, set at $\sigma = 0.40$, $\lambda_\upsilon = 0.976$, $\sigma_\upsilon = 0.0072$ for quarterly data. We agree with the critique (e.g. Summers, 1986; Eichenbaum, 1991) that technological shocks are only a fraction of the Solow residual. The implication is that σ_υ should be a fraction of 0.0072 and accordingly, we set these parameters in our model at $\sigma = 0.40$, $\lambda_\upsilon = 0.976$ and $\sigma_\upsilon = 0.003$. However, for low values of σ_υ the RBC model cannot explain the observed data (see King and Rebelo, 1999, Fig. 8, p. 965), and an alternative propagation mechanism is needed. Examples of such models within the RBC tradition include Wen (1998a, b) and King and Rebelo (1999).

Capacity utilization was studied by writers such as Greenwood et al. (1988), Burnside et al. (1995), Burnside and Eichenbaum (1996), Basu (1996) and others. They show it is an important component of cyclical fluctuations. We agree that under-employed resources are central to economic fluctuations. Indeed, a weak component of our model (which we plan to correct in the future) is its failure to incorporate under-employed labor. In the absence of explicit labor unemployment, capacity utilization in our model should be taken as *a general proxy for factors which can be more intensely utilized when needed.*

Capital accumulation of j is described by a linear transition defined by

$$K^j_{t+1} = (1 - \Delta(\varphi^j_t))K^j_t + I^j_t, \quad \Delta(\varphi_t) = \bar{\delta} + \frac{\delta_0}{\tau}\varphi^\tau_t, \tag{8}$$

where $\Delta(\varphi_t)$ is the rate of depreciation. The empirical evidence about the elasticity τ is mixed. For example, King and Rebelo (1999) use the value $\tau = 1.1$ while Burnside and Eichenbaum (1996) estimate $\tau = 1.56$. We set τ a bit smaller $\tau = 1.3$ to give some

[2]A *Stable Process* is defined in Kurz (1994). It is a stochastic process which has an empirical distribution defined by the limits of relative frequencies of finite dimensional events. These limits are used to define the empirical distribution which, in turn, induces a probability measure over infinite sequences of observables which we refer to as the 'stationary measure' or the 'empirical measure.' A general definition and existence of this probability measure is given in Kurz (1994, 1997a) or Kurz and Motolese (2001) where it is shown that this probability must be stationary. Statements in the text about 'the stationary measure' or 'the empirical distribution' is always a reference to this probability. Its centrality arises from the fact that it is derived from public information and hence *the stationary measure is known to all agents and agreed upon by all to reflect the empirical distribution of observable equilibrium variables.*

[3]The central assumption is then that *agents do not know the true probability but have ample past data from which they deduce that* (2′) *is implied by the empirical distribution.* Hence the data reveals a memory of length 1 and residuals which are i.i.d. normal. This assumption means that even if (2′) is the true data generating process, agents do not know this fact. An agent may believe the true process is nonstationary and different from (2′) and then build his subjective model of the market.

representation to potential under-employed labor which is missing from our model. The availability of under-employed resources which can be mobilized in response to expectations is central to our approach. The other two parameters in (8) are determined by the data. The mean rate of depreciation is 0.025 per quarter and the mean rate of capacity utilization 0.80. Hence we have the implied parameter restriction

$$\bar{\delta} + \frac{\delta_0}{\tau}(0.8)^{\tau} = 0.025.$$

We show later that the last parameter is pinned down by our assumption that the economy has a riskless steady state. I_t^j in (8) is investment of j measuring the number of new units of capital put into production at $t+1$. We normalize

$$k_t^j = \frac{K_t^j}{\xi_t}, \quad i_t^j = \frac{I_t^j}{\xi_t}, \quad w_t = \frac{\tilde{W}_t}{\xi_t P_t}, \quad y_t^j = \frac{Y_t^j}{\xi_t}$$

and define

$$y_t^j = e^{\upsilon_t} L_t^j \left(\frac{\varphi_t^j k_t}{L_t^j} \right)^{\sigma}, \tag{9a}$$

$$k_{t+1}^j = \frac{(1 - \Delta(\varphi_t^j))k_t^j + i_t^j}{\upsilon^*}. \tag{9b}$$

The competitive firms carry out production and investment decisions using the stochastic discount rate $\mu_{t+n,t}^j$ in (7). Gross real capital income of firm j is $Y_t^j - W_t L_t^j$. It incurs real investment cost of I_t^j hence at some dates the net cash flow may be negative. It maximizes

$$\underset{(L_{t+n}^j, I_{t+n}^j, \varphi_{t+n}^j)}{\text{Max}} \; \mathrm{E}_{Q_t^j} \left(\sum_{n=0}^{\infty} \mu_{t+n,t}^j (Y_{t+n}^j - W_{t+n} L_{t+n}^j - I_{t+n}^j) \mid H_t \right) \tag{10}$$

subject to (8), (9a)–(9b). Changes to capacity utilization entail reorganization including engineering design, plans for second shifts, etc. Evidence shows such decisions are carried with delay. We model the planning period for capacity utilization changes to be three months. We thus *assume capacity utilization decisions at* $t+1$ *are made at* t: a firm must commit to a utilization rate one period ahead. Such changes correspond to the process of investments where a firm commits at date t to an investment plan which results in capital employed at date $t+1$. Hence, a commitment to capital and a commitment to a utilization are actually made together as they are naturally joint decisions.

To state the first order conditions define first the *normalized* marginal productivity of factors

$$y_{L_t^j} = e^{\upsilon_t}(1-\sigma) \left(\frac{\varphi_t^j k_t^j}{L_t^j} \right)^{\sigma}, \quad y_{k_t^j} = e^{\upsilon_t} \sigma \left(\frac{\varphi_t^j k_t^j}{L_t^j} \right)^{\sigma - 1} \varphi_t^j.$$

Then the three Euler equations are as follows:

$$0 = y_{L_t^j} - w_t, \tag{10a}$$

$$1 = \mathrm{E}_{Q_t^j}(\mu_{t+1,t}^j(1 + y_{k_{t+1}^j} - \Delta(\varphi_{t+1}^j))), \tag{10b}$$

$$0 = \mathrm{E}_{Q_t^j}(\mu_{t+1,t}^j(y_{k_{t+1}^j} - \delta_0[\varphi_{t+1}^j]^\tau)). \tag{10c}$$

(10c) determines capacity utilization but at steady state we require $(\varphi^j) = 0.80$ hence (10c) imposes the final steady state condition $\beta(\upsilon^*)^{-\gamma}\sigma(0.80k^i/L^j)^{\sigma-1} = \delta_0[0.80]^{\tau-1}$ on the parameters $(\bar{\delta}, \delta_0, \tau)$.

2.3. Monetary policy and money neutrality

In analyzing monetary policy we examine two models. We first explore the simple stochastic exogenous monetary growth with an empirical distribution which is described by the familiar process

$$M_t = M_{t-1}\upsilon^* e^{\varrho_t}, \tag{11}$$

$$\varrho_{t+1} = \lambda_\varrho \varrho_t + \rho_{t+1}^\varrho. \tag{12}$$

Since $m_t = M_t/(\xi_t P_t)$, we have $m_t = m_{t-1}e^{(\varrho_t - \pi_t)}$. The model of money injection is a bit unrealistic since it is hard to see a central bank distributing money to cash holders when $\varrho_t > 0$ and extracting it when $\varrho_t < 0$. Also, since there is no one 'money,' which of the near moneys should a bank control? In spite of these drawbacks the monetary shocks model is a useful idealization. It provides a *reference point to measure the efficacy of monetary policy in economies with a monetary rule*.

We next study economies with nominal interest rate rule. A central bank sets the one period nominal rate r_t. The inflation rate target is assumed 1% per quarter. In the simulations we consider the performance of Taylor (1993) type policy rules with and without discretion and inertia. Discretion introduces into the policy rule a random component d_t which will be assumed to have an empirical distribution which is analogous to (12)

$$d_{t+1} = \lambda_d d_t + \rho_{t+1}^d. \tag{13}$$

Details are presented and discussed in Section 5. The assumption of a balanced budget under the proposed policy rule requires the lump-sum tax or subsidy rates to satisfy $m_t - (m_{t-1}/e^{\pi_t}\upsilon^*) = T_t$.

A Comment On Money Neutrality. In a model with monetary shocks, date $t+1$ shock is not known at t. But once realized, agents observe them and equilibrium prices adjust to neutralize them. This means that although unanticipated monetary shocks are included in the model, even under REE such unanticipated shocks have no real effects. Hence, our model is *strongly biased in favor of money neutrality*. However, this mechanism does not work in an RBE where agents expect money shocks to have real effect but disagree about the magnitude of the effects. In an RBE agents form beliefs about the future course of the economy and this includes the real effects of money shocks. The crucial channel for money nonneutrality are the diverse belief of agents *about future beliefs of other agents in the market*. Indirectly, this implies diverse beliefs about the real effects of money shocks.

2.4. Market clearing conditions

The market clearing conditions are then

$$b_t^1 + b_t^2 = 0 \quad \text{for all } t; \tag{14a}$$

$$(1 - \ell_t^1) + (1 - \ell_t^2) = L_t^1 + L_t^2 \quad \text{for all } t; \tag{14b}$$

$$\text{for the money shock model} \quad m_t^1 + m_t^2 = m_t \quad \text{for all } t; \tag{14c}$$

$$\text{for the interest rule model} \quad m_t - \frac{m_{t-1}}{e^{\pi_t} \upsilon^*} = T_t \quad \text{for all } t. \tag{14d}$$

We now turn to the central question of this paper which is the diverse beliefs of the agents.

3. The general structure of equilibria with diverse and time dependent beliefs

The economy of this paper has nonstationary dynamics and is populated by agents with time dependent conditional probability beliefs. These are complex economies and a general treatment is difficult. To simplify we study monetary policy by *constructing a family of equilibria* defined by an autoregressive stochastic law of motion of the state variables. Also, results of our paper apply to agents who hold RB. We must, however, distinguish between the general structure of equilibria with diverse and time varying beliefs, and the restrictions on beliefs imposed by RB rationality.

The general structure is applicable to situations with similar dynamics. For example, in an economy in which agents do not know some parameters they use diverse priors or diverse learning rules to deduce them from the data. As they learn parameters, their beliefs vary. A second example are economies with varying, unobserved, production regimes and diverse beliefs about the state of such regimes. The general structure does not specify restrictions on beliefs. An RBE, however, is an equilibrium in which the RB rationality principle is imposed on the beliefs of agents and we later explain these restrictions.

3.1. Market states of belief and anonymity: expansion of the state space

Although (2′) and (12) represent all the moments of past data, agents do not believe a fixed stationary model captures the complexity of the economy. Also, they do not generally agree on one 'correct' model that generated the empirical evidence in (2′) and (12). Indeed, we expect that agents using the same evidence will come up with different theories to explain the data and hence with different models to forecast prices. But then, one may ask, what are the specific formal belief formation models agents use to deviate from the empirical forecasts? Since they do not hold rational expectations, why do they select these belief formation models? These are questions which we do not address here. Our methodology is *to use distributions of belief to explain market volatility* since we view belief diversity as important a primitive of the economy as utilities and endowments. Hence, we need to determine a level of detail at which agents 'justify' their beliefs. If we aim at a complete specification of such models, our study is doomed to be bogged down in details of information processing. Although interesting, from an equilibrium perspective it is not needed: the reasoning that lead agents to their subjective models are secondary. To study monetary policy we focus on a narrow but operational problem. Since Euler equations require specification of conditional probabilities, we only need a tractable way to *describe* the differences among agents' beliefs and the time variability of their conditional probabilities. This statistical regularity is crucial for determining if they satisfy the rationality of belief conditions. As in Kurz et al. (2005), the tool we developed for this goal is the individual and the market '*state of belief*' which we now explain.

The usual state space for agent j is denoted by S^j but when beliefs change we introduce an additional state variable called '*agent j state of belief*.' It is a parameter generated by agent j and is denoted by $g_t^j \in G^j$. It has the property that once specified, the conditional probability function of an agent is uniquely specified hence has the form $\Pr(s_{t+1}^j, g_{t+1}^j \mid s_t^j, g_t^j)$. The parameter g_t^j is thus a proxy for j's conditional probability function and g_t^j is *privately* observed by agent j. Since a dynasty consists of a sequence of decision makers, we assume g_t^j is known to j but is not observed by members who follow him. In the present model agents forecast productivity growth rate hence $g_t^j \in \mathbb{R}$ describes agent j conditional probability of productivity growth at $t+1$. We permit agents to be optimistic at t and expect above normal (measured by the empirical distribution) productivity growth at date $t+1$ or pessimistic and expect below normal date $t+1$ productivity growth. g_t^j is centered around 0 and we interpret g_t^j in the following way:

- If $g_t^j = 0$ agent j believes that all empirical distributions in the model are the true processes and hence makes productivity growth forecasts in accord with (2′);
- If $g_t^j \neq 0$ agent j disagrees with the empirical distributions. If $g_t^j > 0$ he is optimistic and makes *higher* productivity growth forecasts than the ones implied by (2′); if $g_t^j < 0$ he is pessimistic and makes *lower* productivity growth forecasts than the ones implied by (2′).

It is a common practice among forecasters to use the strict econometric forecast only as a benchmark. Given a benchmark, a forecaster uses his own subjective model to add a component reflecting an evaluation of the circumstances at a date t which call for deviation from the benchmark. In this paper we assume that the state of belief is a date t realization of a process of the form

$$g^j_{t+1} = \lambda_z g^j_t + \lambda^z_v v_t + \lambda^z_\varrho \varrho_t + \tilde{\rho}^{g^j}_{t+1}, \quad \tilde{\rho}^{g^j}_{t+1} \sim \mathrm{N}(0, \tilde{\sigma}^2_{g^j}), \quad j = 1, 2. \tag{15}$$

Persistent states of belief which depends upon fundamental variables fit different economies with diverse beliefs. We consider three examples to illustrate how one may think about them.

(i) *Measure of animal spirit*. 'Animal Spirit' expresses judgment and intensity of investment decision and this, in turn, is based upon expected rewards. g^j_t identifies the probability an agent assigns to high or low rates of return hence g^j_t can be interpreted as a measure of 'animal spirit.'

(ii) *Learning unknown parameters*. When learning, agents use prior distributions on unknown parameters. If g^j_t defines a belief about a variable v_{t+1} we can identify g^j_{t+1} as a posterior parameter of v_{t+1}. A posterior as a linear function of a prior and current data is familiar. We add $\tilde{\rho}^{g^j}_{t+1}$ to reflect changes in priors due to change in structure over time. Such changes includes changes in dynasty decision makers. With diverse beliefs, $\tilde{\rho}^{g^j}_{t+1}$ can model a diversity of prior beliefs over time.

(iii) *Privately generated subjective sunspot which depend upon real variables*. g^j_t may be a private sunspot with three properties (a) an agent generates his own g^j_t under a marginal distribution known only to him, (b) g^j_t is not observed by other agents, and (c) it's distribution may depend upon real variables. In addition, the correlation across agents is a market externality, not known to anyone. Under this interpretation g^j_t is a major extension of the common concept of a 'sunspot' variable.

In equilibria with diverse beliefs agents' decisions are functions of g^j_t hence prices depend upon $g_t = (g^1_t, g^2_t, \ldots, g^N_t)$. But then, should j be allowed to recognize that his g^j_t is the jth coordinate of g_t, giving him market power? The principle of *anonymity* requires competitive agents to assume they cannot affect endogenous variables. It is analogous to competitive firms who assume they have no effect on prices. We define the '*market state of belief*' to be $z_t = (z^1_t, z^2_t, \ldots, z^N_t)$, with internal consistency condition $z_t = g_t$ *which is not recognized by agents*. This makes the market state of belief a macroeconomic state variable hence prices are actually functions of z_t and not of g_t.

Agent j views z_t as a market belief and unrelated to him since it is the belief of *other agents*. In small economies prices depend upon the distribution $z_t = (z^1_t, z^2_t, \ldots, z^N_t)$ but in many applications only a few moments matter. In some models writers focus only on the average, defining the market state of belief by the mean belief[4] $z_t = 1/N \sum_{j=1}^N z^j_t$. We can simplify the theory greatly by assuming that $z_t = (z^1_t, z^2_t, \ldots, z^N_t)$ is observable and we argue later that this assumption is empirically justified.

Anonymity is so central to our approach that we use three notational devices to highlight it:

(i) g^j_t denotes the state of belief of j as known and observed only by the agent.
(ii) $z_t = (z^1_t, z^2_t, \ldots, z^N_t)$ denotes market state of belief, observed by all.

[4]See Woodford (2003a), Morris and Shin (2002), Allen et al. (2003) and others.

(iii) $z^j_{t+1} = (z^{j1}_{t+1}, z^{j2}_{t+1}, \ldots, z^{jN}_{t+1})$ is agent j's perception of the market state of belief at future date $t+1$.

The introduction of individual and market states of belief has two central implications:

(A) The economy has an *expanded state space*, including z_t. $z_t = (z^1_t, z^2_t) \in \mathbb{R}^2$ in this paper. Hence diverse beliefs create new uncertainty which is the uncertainty of *what others will do.* This adds a component of volatility which cannot be explained by 'fundamental' shocks. Denoting usual state variables by s_t, the price process $\{(q^b_t, w_t, \pi_t, T_t), t = 1, 2, \ldots\}$ is then defined by a map like

$$\begin{bmatrix} q^b_t \\ w_t \\ \pi_t \\ T_t \end{bmatrix} = \Xi(s_t, z^1_t, z^2_t, \ldots, z^N_t). \tag{16}$$

Our equilibrium will thus be an incomplete Radner (1972) equilibrium with *an expanded state space.*

(B) To forecast prices agents must forecast market beliefs. Although all use (16) to forecast prices, agents' forecasts are different since an agent forecasts (s_{t+1}, z_{t+1}) *given his own state* g^j_t. This is a feature of the Keynes Beauty Contest: in order to forecast equilibrium prices you must forecast the beliefs of other agents. A Beauty Contest *does not entail higher order of beliefs*: at t you form belief about market belief z_{t+1} but the $t+1$ market belief *is not* a probability about your date t belief state.

We now return to the economy with two agent types and simplify by assuming (z^1_t, z^2_t) *is observable*. This assumption is reasonable since there is a vast amount of data on the distribution of market forecasts. Indeed, using data from the Blue Chip Economic Indicators and the Survey of Professional Forecasters we constructed various measures of market states of belief. Since (z^1_t, z^2_t) is assumed observable we must modify the empirical distribution (2′)–(12) to include (z^1_t, z^2_t) in it. Recall that the true process of the shocks $\{(\upsilon_t, \varrho_t), t = 1, 2, \ldots\}$ is an unspecified, stable and nonstationary process. Indeed, for equilibrium analysis the true process does not matter: what matters is the empirical distribution of the process and what agents believe about the true process. Our state variables are $(\upsilon_t, \varrho_t, z^1_t, z^2_t)$ and we simplify by selecting the joint empirical distribution to be a stationary transition which is an AR process of the form

$$\begin{aligned}
\upsilon_{t+1} &= \lambda_\upsilon \upsilon_t + \rho^\upsilon_{t+1} \\
\varrho_{t+1} &= \lambda_\varrho \varrho_t + \rho^\varrho_{t+1} \\
z^1_{t+1} &= \lambda_{z^1} z^1_t + \lambda^{z^1}_\upsilon \upsilon_t + \lambda^{z^1}_\varrho \varrho_t + \rho^{z^1}_{t+1}, \\
z^2_{t+1} &= \lambda_{z^2} z^2_t + \lambda^{z^2}_\upsilon \upsilon_t + \lambda^{z^2}_\varrho \varrho_t + \rho^{z^2}_{t+1}
\end{aligned}$$

$$\begin{pmatrix} \rho_t^{\upsilon} \\ \rho_t^{\varrho} \\ \rho_t^{z^1} \\ \rho_t^{z^2} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\upsilon}^2, & 0, & 0, & 0 \\ 0, & \sigma_{\varrho}^2, & 0, & 0 \\ 0, & 0, & 1, & \sigma_{z^1 z^2} \\ 0, & 0, & \sigma_{z^1 z^2}, & 1 \end{pmatrix} \right) \text{ i.i.d.} \tag{17}$$

In constructing an equilibrium our theory assumes (17) is known to all. We have already noted that $\lambda_{\upsilon} = 0.976$, $\sigma_{\upsilon} = 0.003$. Money shock parameters are taken from Mankiw and Reis's (2002) who estimate $\lambda_{\varrho} = 0.5$, $\sigma_{\varrho} = 0.007$. To specify the z^j equations parameters we used forecasts of the Survey of Professional Forecasters and Blue Chip Indicators and 'purged' them of observables. By estimating principal components we handle multiple forecasted variables (for details, see Fan, 2004). The extracted belief indexes imply regression coefficients of 0.5–0.8 and since we assume symmetry across agents we set $\lambda_{z^1} = \lambda_{z^2} = \lambda_z = 0.65$. Evidence reveals high correlation of forecasted macro economic variables across forecasters and 0.9 is a reasonable estimate of $\sigma_{z^1 z^2}$.

We study symmetric economies with $\lambda_{\upsilon}^{z^1} = \lambda_{\upsilon}^{z^2} = \lambda_{\upsilon}^{z}$ and $\lambda_{\varrho}^{z^1} = \lambda_{\varrho}^{z^2} = \lambda_{\varrho}^{z}$ hence agents differ only in the state of their belief. Evidence reveals that above normal productivity shocks lead to common upward revisions of the mean growth rate, implying $\lambda_{\upsilon}^{z} > 0$. In the simulations we set $\lambda_{\upsilon}^{z} = 8$ but this has small impact. We have little evidence on λ_{ϱ}^{z} which has important money nonneutrality impact. We set $\lambda_{\varrho}^{z} = 8$ with a simple empirical reasoning to support it: *for positive money shocks to induce positive impulse response of output and consumption* $\lambda_{\varrho}^{z} > 0$ *is needed.* Positive money shocks at t lead agents to expect above normal z_{t+1}^j and hence, above normal future output level.

To write (17) in a compact notation let $x_t = (\upsilon_t, \varrho_t, z_t^1, z_t^2)$, $\rho_t = (\rho_t^{\upsilon}, \rho_t^{\varrho}, \rho_t^{z^1}, \rho_t^{z^2})$ and denote by A the 4×4 matrix of parameters in (17). We then write

$$x_{t+1} = Ax_t + \rho_{t+1}, \quad \rho_{t+1} \sim \mathrm{N}(0, \Sigma). \tag{18}$$

Σ is the covariance matrix in (17). Let Γ be the probability measure on infinite sequences implied by (17) with the invariant distribution as an initial distribution. Hence we write $\mathrm{E}_{\Gamma}(x_{t+1} \mid H_t) = Ax_t$ where H_t is the history at t. In addition, V is the 4×4 *unconditional* covariance matrix of x defined by $V = \mathrm{E}_{\Gamma}(xx')$. From (18) it follows that V is the solution of the equation

$$V = AVA' + \Sigma. \tag{18a}$$

To complete the description of an equilibrium we now turn to the belief structure.

3.2. The general structure of beliefs and the problem of parameters

A *perception model* is a set of transition functions of state variables, expressing an agent's conditional probability belief. Let $x_{t+1}^j = (\upsilon_{t+1}^j, \varrho_{t+1}^j, z_{t+1}^{1j}, z_{t+1}^{2j})$ be date $t+1$ variables *as perceived by j* and let $\Psi_{t+1}(g_t^j)$ be a four dimensional vector of date $t+1$ random variables *conditional upon* g_t^j.

Definition 1. A perception model in the economy under study has the general form

$$x^j_{t+1} = Ax_t + \Psi_{t+1}(g^j_t) \text{ together with (15).} \tag{19a}$$

Since $\mathrm{E}_\Gamma(x_{t+1} \mid H_t) = Ax_t$, we write (19a) in the simpler form

$$x^j_{t+1} - \mathrm{E}_\Gamma(x_{t+1} \mid H_t) = \Psi_{t+1}(g^j_t). \tag{19b}$$

In general, $\mathrm{E}^j_t[\Psi_{t+1} \mid g^j_t] \neq 0$ hence agent's forecast function changes with g^j_t. If $\Psi_{t+1}(g^j_t) = \rho_{t+1}$ as in (17), j uses the empirical probability Γ as his belief. Condition (19b) shows that we model $\Psi_{t+1}(g^j_t)$ so that agents may be over-confident by being optimistic or pessimistic *relative to the empirical forecasts*. We now postulate *a single random variable* $\eta^j_{t+1}(g^j_t)$ with which we model all components of the functions $\Psi_{t+1}(g^j_t)$, taking the following form:

$$\Psi_{t+1}(g^j_t) = \begin{pmatrix} \lambda^{\upsilon}_g \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{\upsilon^j}_{t+1} \\ \lambda^{\varrho}_g \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{\varrho^j}_{t+1} \\ \lambda^{z1}_g \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{z^{j1}}_{t+1} \\ \lambda^{z2}_g \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{z^{j2}}_{t+1} \end{pmatrix}, \quad \tilde{\rho}^j_{t+1} \sim \mathrm{N}(0, \Omega^j_{\eta\eta}) \text{ i.i.d.} \tag{20}$$

where $\tilde{\rho}^j_{t+1} = (\tilde{\rho}^{\upsilon^j}_{t+1}, \tilde{\rho}^{\varrho^j}_{t+1}, \tilde{\rho}^{z^{j1}}_{t+1}, \tilde{\rho}^{z^{j2}}_{t+1})$. By (19a) a perception model includes g^j_{t+1} as a fourth dimension with innovation $\tilde{\rho}^j_{t+1}$ and a covariance matrix denoted Ω^j, including the vector $r^i_j = \mathrm{Cov}(x^i, g^j)$ for $i = 1, 2, 3, 4$. Also, we study symmetric markets hence assume $\lambda^z_g = \lambda^{z1}_g = \lambda^{z2}_g$ and $\lambda_g = (\lambda^{\upsilon}_g, \lambda^{\varrho}_g, \lambda^z_g)$.

To specify an agents' beliefs we specify $\Psi_{t+1}(g^j_t)$, $\lambda_g = (\lambda^{\upsilon}_g, \lambda^{\varrho}_g, \lambda^z_g)$ and $r^i_j = \mathrm{Cov}(x^i, g^j)$ for $i = 1, 2, 3, 4$. Combining all the parts we formulate the perception model of agent j by

$$\begin{aligned}
\upsilon^j_{t+1} &= \lambda_\upsilon \upsilon_t + \lambda^{\upsilon}_g \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{\upsilon^j}_{t+1}, \\
\varrho^j_{t+1} &= \lambda_\varrho \varrho_t + \lambda^{\varrho}_g \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{\varrho^j}_{t+1}, \\
z^{j1}_{t+1} &= \lambda_z z^1_t + \lambda^z_\upsilon \upsilon_t + \lambda^z_\varrho \varrho_t + \lambda^z_g \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{z^{j1}}_{t+1}, \\
z^{j2}_{t+1} &= \lambda_z z^2_t + \lambda^z_\upsilon \upsilon_t + \lambda^z_\varrho \varrho_t + \lambda^z_g \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{z^{j2}}_{t+1}, \\
g^j_{t+1} &= \lambda_z g^j_t + \lambda^z_\upsilon \upsilon_t + \lambda^z_\varrho \varrho_t + \tilde{\rho}^{g^j}_{t+1}.
\end{aligned} \tag{21a}$$

$\tilde{\rho}^j_{t+1} = (\tilde{\rho}^{\upsilon^j}_{t+1}, \tilde{\rho}^{\varrho^j}_{t+1}, \tilde{\rho}^{z^{j1}}_{t+1}, \tilde{\rho}^{z^{j2}}_{t+1}, \tilde{\rho}^{g^j}_{t+1})$ is i.i.d. Normal with mean zero and covariance matrix Ω^j of the form

$$\Omega^j = \begin{pmatrix} \Omega^j_{\rho\rho}, \Omega_{xg^j} \\ \Omega_{xg^j}, \sigma^2_{g^j} \end{pmatrix}, \tag{21b}$$

where $\Omega_{xg^j} = [\mathrm{Cov}(\tilde{\rho}^{\upsilon^j}_{t+1}, \tilde{\rho}^{g^j}_{t+1}), \mathrm{Cov}(\tilde{\rho}^{\varrho^j}_{t+1}, \tilde{\rho}^{g^j}_{t+1}), \mathrm{Cov}(\tilde{\rho}^{z^{j1}}_{t+1}, \tilde{\rho}^{g^j}_{t+1}), \mathrm{Cov}(\tilde{\rho}^{z^{j2}}_{t+1}, \tilde{\rho}^{g^j}_{t+1})]$. We note that $\lambda^z_g \eta_{t+1}(g^j_t)$ in the third and fourth equations measure the effect of j's belief *on his forecast of the belief of others* $(z^{j1}_{t+1}, z^{j2}_{t+1})$ at $t+1$. In Section 6 we show these are central to the efficacy of monetary policy and to the violation of iterated expectations of the average market belief. Finally, (21a)–(21b) show that $\lambda_g = (\lambda^v_g, \lambda^\varrho_g, \lambda^z_g) = (0,0,0)$ characterizes an economy where agents believe the empirical distribution is the truth. Without diverse beliefs, such economy has the key property of an REE.

Summary of belief parameters. The 'free' parameters specifying the belief of j are (λ_g, Ω^j) together with $\eta^j_{t+1}(g^j_t)$ (explained next). A*nonymity* requires the idiosyncratic component of an agent's belief not to be correlated with any market beliefs. This is translated into the requirement that

$$\Omega_{z^1 g^j} = \mathrm{Cov}(\tilde{\rho}^{z^{j1}}_{t+1}, \tilde{\rho}^{g^j}_{t+1}) = 0, \tag{22a}$$

$$\Omega_{z^2 g^j} = \mathrm{Cov}(\tilde{\rho}^{z^{j2}}_{t+1}, \tilde{\rho}^{g^j}_{t+1}) = 0. \tag{22b}$$

Hence, anonymity restricts two components of Ω_{xg^j} even in a model without restrictions on belief.

3.3. Modeling tractable and computable random functions $\Psi_{t+1}(g^j_t)$

We model $\Psi_{t+1}(g^j_t)$ so as to permit agents to be *over confident* by assigning to some events higher or lower probability than the empirical frequency. Evidence from psychology (e.g. Svenson, 1981; Camerer and Lovallo, 1999 and references there) shows agents exhibit such behavior. In some cases this behavior is irrational but this need not be generally true. Changes are central to an economy and past statistics may not provide the best forecasts for the future. Hence, deviations from empirical frequencies reflect views based on limited *recent* data about changed conditions. Also, agents have strong financial incentive to make such judgments since *major financial gains are available to those who bet on the correct deviation of the economy from the empirical frequency*.

The random variables $\Psi_{t+1}(g^j_t)$: *intensity of fat tails in an agent's belief*. 'Fat' tails, reflecting over confidence, is introduced into the computational model through $\eta^j_{t+1}(g^j_t)$. We define $\eta^j_{t+1}(g^j_t)$ by its density, conditional on g^j_t, as a step function

$$p(\eta^j_{t+1}|g^j) = \begin{cases} \psi_1(g^j)\Phi(\eta^j_{t+1}) & \text{if } \eta^j_{t+1} \geqslant 0, \\ \psi_2(g^j)\Phi(\eta^j_{t+1}) & \text{if } \eta^j_{t+1} < 0, \end{cases} \tag{23}$$

where η^j_{t+1} and $\tilde{\rho}^{g^j}_{t+1}$ (in (20)) are independent and where $\Phi(\eta) = [1/\sqrt{2\pi}]e^{-\eta^2/2}$. The functions $(\psi_1(g), \psi_2(g))$ are defined by a logistic function with a single parameter b

$$\psi(g^j) = \frac{1}{1+e^{bg^j}}, \quad b<0, \quad G \equiv \mathrm{E}_g[\psi(g^j)],$$

$$\psi_1(g^j) = \frac{\psi(g^j)}{G}, \quad \psi_2(g^j) = 2 - \psi_1(g^j). \tag{24}$$

The parameter b measures *intensity of fat tails* in beliefs. Details of this construction and the implied moments are discussed in detail in Appendix A.

Eqs. (23)–(24) are a formal description of over confidence – relative to the empirical distribution – via η^j_{t+1}. For g^j_t large, $\psi(g^j)$ goes to one, implying that $\psi_1(g^j)$ goes to $1/G$. Hence, by (23) large $g^j_t > 0$ implies high probability of $\eta^j_{t+1} > 0$. Similarly, small $g^j_t < 0$ implies high probability of $\eta^j_{t+1} < 0$. These show our interpretation of $g^j_t > 0$ amounts to *a model convention* requiring $b < 0$ and now needing to be implemented in $\Psi_{t+1}(g^j_t)$. To that end note that if $\lambda^{\upsilon}_g > 0$, a positive value of $\eta^j_{t+1} > 0$ increases j's forecast of υ^j_{t+1}while in an economy with $\lambda^{\upsilon}_g < 0$ a value of $\eta^j_{t+1} > 0$ lowers j's forecast of υ^j_{t+1}. This leads to a formal definition of what $g^j_t > 0$ means in terms of over confidence:

Definition 2. Let Q^j be the probability belief of agent j. Then g^j_t is agent j's state of *over confidence in abnormally high productivity growth* if $E^j_t[\upsilon^j_{t+1}|g^j_t, H_t] > E_{\Gamma}(\upsilon_{t+1}|H_t)$;
over confidence in abnormally low productivity growth if $E^j_t[\upsilon^j_{t+1}|g^j_t, H_t] < E_{\Gamma}(\upsilon_{t+1}|H_t)$.

For brevity we refer to these forms of over confidence as 'optimism' and 'pessimism.' From (21a) we know that $E^j_t[\upsilon^j_{t+1}|g^j_t, H_t] - E_{\Gamma}(\upsilon_{t+1}|H_t) = \lambda^{\upsilon}_g E^j_t[\eta^j_{t+1}(g^j_t)|g^j_t]$. In Appendix A we show that $E^j_t[\eta^j_{t+1}(g^j_t)|g^j_t > 0] > 0$ and $E^j_t[\eta^j_{t+1}(g^j_t)|g^j_t < 0] < 0$. Together, they lead to the conclusion that the model convention we have adopted, regarding the interpretation of $g^j_t > 0$, requires $\lambda^{\upsilon}_g > 0$.

How do we then describe the beliefs? As g^j_t increase, $\psi_1(g^j_t)$ rises and $\psi_2(g^j_t)$ falls. Hence when $g^j_t > 0$ an agent raises the positive part of a normal density in (23) by a factor $\psi_1(g^j_t) > 1$ and reduces the negative part by $\psi_2(g^j_t) < 1$. When $g^j_t < 0$ the opposite occurs: the *negative* part shifts up by $\psi_2(g^j_t) > 1$ and the positive part shifts down by $\psi_1(g^j_t) < 1$. The amplifications $(\psi_1(g^j), \psi_2(g^j))$ are defined by g^j and by the 'fat tails' parameter b which measures the degree by which the distribution shifts per unit of g^j_t. In Fig. 1 we draw densities of $\eta^j(g^j)$ for $g^j > 0$ and for $g^j < 0$. *These are not normal densities.* As g^j_t varies, the densities of $\eta^j_{t+1}(g^j_t)$ change. However, the empirical distribution of g^j_t is normal with zero unconditional mean and hence the empirical distribution of $\eta^j_{t+1}(g^j_t)$, averaged over time (including over g^j_t) , also has these same properties.

Each component of $\Psi_{t+1}(g^j_t)$ is a sum of two random variables: one as in Fig. 1 and the second is normal. In Fig. 2 we draw two densities of the υ component of $\Psi_{t+1}(g^j_t)$, each being a convolution of the two constituent distributions with $\lambda^{\upsilon}_g > 0$. One density for $g^j > 0$ and a second for $g^j < 0$, both having 'fat tails.' Since b measures intensity by which the positive portion of the distribution in Fig. 1 is shifted, it measures the degree of fat tails in the distributions of $\Psi_{t+1}(g^j_t)$.

3.4. Restrictions on beliefs in an RBE under the rational belief principle

We now define a rational belief (due to Kurz, 1994, 1996) and discuss the restrictions which the theory imposes on the belief parameters of the agents in our model above.

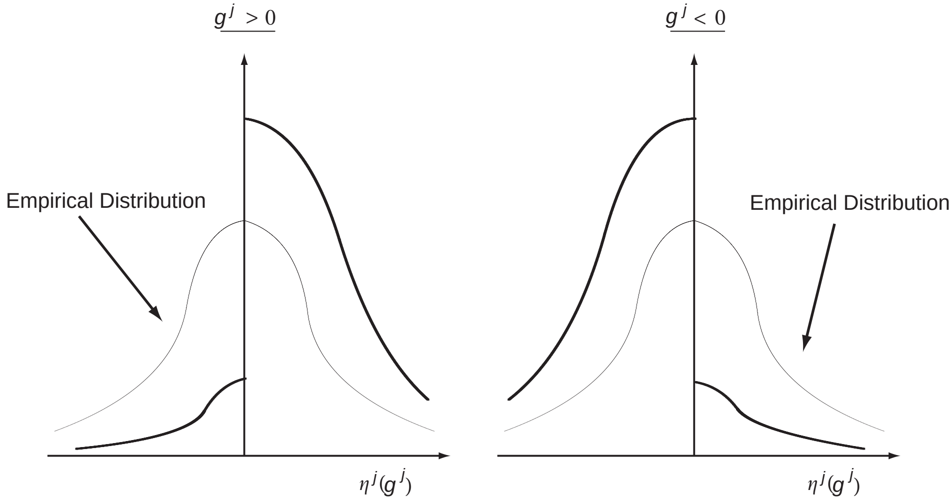

Fig. 1. non-normal belief densities.

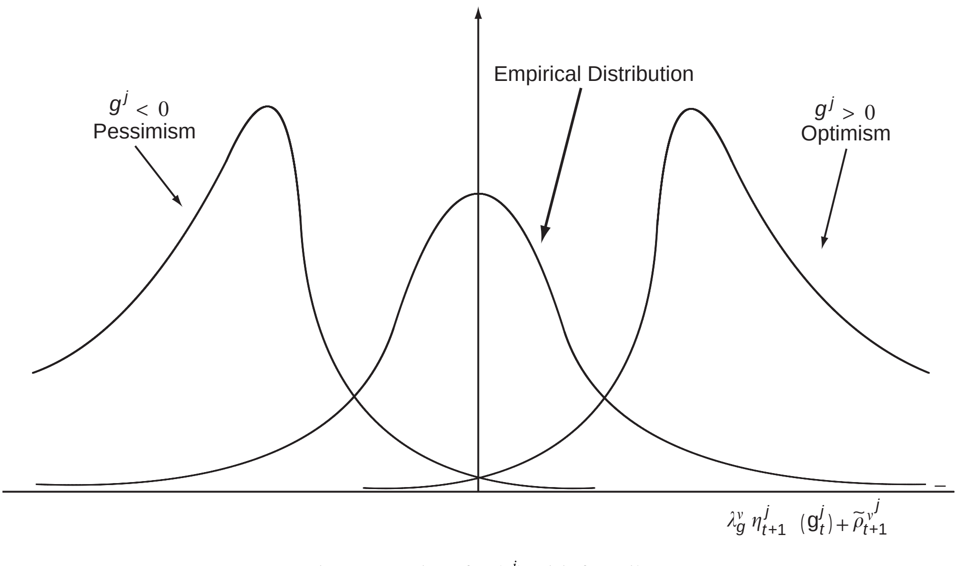

Fig. 2. Density of $\Psi(g_t^j)$ with fat tails.

Definition 3. A perception model as defined in (21a)–(21b) is a rational belief if the agent's model $x_{t+1}^j = Ax_t + \Psi_{t+1}(g_t^j)$ together with (15) has the same empirical distribution as $x_{t+1} = Ax_t + \rho_{t+1}$ in (18).

Definition 3 implies that $\Psi_{t+1}(g_t^j)$ together with (15) must have the same empirical distribution as ρ_{t+1} in (18), i.e. $\mathrm{N}(0, \Sigma)$. An RB is a model which cannot be rejected by the data as it matches *all moments of the observables*. Agents holding RB may exhibit over confidence by deviating from the empirical frequencies but their behavior is rationalizable if the time average of the probabilities of an event equals it's empirical frequency. What are the restrictions implied by the RB principle?

Theorem. *Let the beliefs of an agent be a rational belief. Then it is restricted as follows*:

(i) *For any feasible vector of parameters* $(\lambda_g^{\upsilon}, \lambda_g^{\varrho}, \lambda_g^{z}, b)$ *the variance– covariance matrix* Ω^j *is fully defined and is not subject to choice*;
(ii) Ω^j *must be a positive definite matrix. This requirement establishes a feasibility region for the vector* $(\lambda_g^{\upsilon}, \lambda_g^{\varrho}, \lambda_g^{z}, b)$. *In particular it requires* $|\lambda_g^{\upsilon}| \leqslant \sigma_{\upsilon}$, $|\lambda_g^{\varrho}| \leqslant \sigma_{\varrho}$, $|\lambda_g^{z}| \leqslant 1$.
(iii) $\{\Psi_{t+1}(g_t^j)\}$ *cannot exhibit serial correlation and this restriction pins down the vector*

$$\Omega_{xg^j} = [\mathrm{Cov}(\tilde{\rho}_{t+1}^{\upsilon^j}, \tilde{\rho}_{t+1}^{g^j}), \mathrm{Cov}(\tilde{\rho}_{t+1}^{\varrho^j}, \tilde{\rho}_{t+1}^{g^j}), \mathrm{Cov}(\tilde{\rho}_{t+1}^{z^{j1}}, \tilde{\rho}_{t+1}^{g^j}), \mathrm{Cov}(\bar{\rho}_{t+1}^{z^{j2}}, \bar{\rho}_{t+1}^{g^j})].$$

Proofs are in Appendix B. Since $\{g_t^j, t = 1, 2, \ldots\}$ exhibit serial correlation, to isolate the pure belief we exclude from g_t^j information in the market at t. We define a *pure belief* index $u_t^j(g_t^j)$. Recall $r_j = \mathrm{Cov}(x, g^j)$ is agent j's covariances and using (18a) define $u_t^j(g_t^j)$ by a standard regression filter

$$u_t^j(g_t^j) = g_t^j - r_j V^{-1} x_t. \tag{25}$$

The index $u_t^j(g_t^j)$ now replaces g_t^j *everywhere* and is uncorrelated with public information. In all equations we replace $\Psi_{t+1}(g_t^j)$ with $\Psi_{t+1}(u_t^j)$ and show in Appendix B *that it is serially uncorrelated.*

Under the RB restrictions we can thus select only $(\lambda_g^{\upsilon}, \lambda_g^{\varrho}, \lambda_g^{z}, b)$ subject to the feasibility conditions imposed by the Theorem. In practice these restrictions imply the following conditions:

- $\sigma_{\upsilon} = 0.003$ implies $|\lambda_g^{\upsilon}| < 0.003$. The covariance structure further restricts $|\lambda_g^{\upsilon}| < 0.0027$.
- $\sigma_{\varrho} = 0.007$ implies $|\lambda_g^{\varrho}| < 0.007$. The covariance structure further restricts $|\lambda_g^{\varrho}| < 0.0032$.
- The covariance structure implies that $|\lambda_g^{z}| < 0.35$.
- The overconfidence parameter b has a feasible range between 0 and -12.

Given our convention we study $\lambda_g^{\upsilon} > 0$ subject to $|\lambda_g^{\upsilon}| < 0.003$. Furthermore, in models of money shocks we assume $\lambda_g^{\varrho} = 0.00$, postulating all agents believe the money growth (12) is the true monetary shock model of the Fed. We adopt a different approach in the case of central bank discretionary shocks. We finally offer some additional considerations to restrict the parameter λ_g^{z}.

3.4.1. Selecting λ_g^z: the principle of enhancing relative market position

We assumed nothing regarding belief of agents about the beliefs of others and have only limited data on it. Let $\bar{z}_t = 1/N \sum_{j=1}^N z_t^j$ be the mean market belief and we ask the following question. Suppose an agent is optimistic about productivity growth. How would this affect his belief about the mean market belief? We could introduce a second belief index to pin down a belief about 'others' to forecast $g_{t+1}^k - \bar{z}_{t+1}^k = g_{t+1}^k - 1/N \sum_{j=1}^N z_{t+1}^{jk}$. But now suppose that, in addition, he is more optimistic than the average so $g_t^j > \bar{z}_t$. How would his optimistic view of productivity alter the expected *relative* position of his belief in relation to the mean market belief? There is no clear answer to this question but data suggests *a form of a relative inertia* which is captured by the following concept:

Definition 4. Agent j expects to *Enhance* his relative position within the belief distribution given his current state of belief if his belief about others satisfies the conditions

$$E_t^j(g_{t+1}^j - \bar{z}_{t+1}^j) > \lambda_z(g_t^j - \bar{z}_t^j) \quad \text{if } g_t^j > 0 \text{ (i.e. when } j \text{ is in an optimistic state);} \tag{26a}$$

$$E_t^j(g_{t+1}^j - \bar{z}_{t+1}^j) < \lambda_z(g_t^j - \bar{z}_t^j) \quad \text{if } g_t^j < 0 \text{ (i.e. when } j \text{ is in a pessimistic state).} \tag{26b}$$

But then recall that by (21a)

$$E_t^j(g_{t+1}^j - \bar{z}_{t+1}^j) - \lambda_z(g_t^j - \bar{z}_t^j) = -\lambda_g^z E_t^j[\eta_{t+1}^j(g_t^j)] \tag{27a}$$

and Appendix B shows that

$$E_t^j[\eta_{t+1}^j(g_t^j)] > 0 \quad \text{if } g_t^j > 0 \quad \text{and} \quad E_t^j[\eta_{t+1}^j(g_t^j)] < 0 \quad \text{if } g_t^j < 0. \tag{27b}$$

Conclusion. Eqs. (26a)–(26b) can occur only if $\lambda_g^z < 0$. Since we adopt (26), we must assume $\lambda_g^z < 0$.

To explain (26a)–(26b) suppose an agent is optimistic and the empirical frequency predicts his relative position at $t+1$ will be $\lambda_z(g_t^j - \bar{z}_t^j)$. Then (26a) says an optimistic state is translated into a prediction in the persistence of his *relative* optimistic belief. Under our assumptions, convention and the condition in (26a)–(26b), the belief parameter must take the following sign pattern:

$$\lambda_g^v > 0, \quad \lambda_g^\varrho = 0, \quad \lambda_g^z < 0, \quad b < 0.$$

The immediate questions we ask is then simple: can we find feasible parameter values so the model replicates the empirical record of the U.S. economy? If yes, we shall use it as the reference economy for our study, in which fluctuations are propagated in part by the beliefs of agents.

3.4.2. Model parameters and note on computational procedure

The parameters $(\lambda_g^v, \lambda_g^\varrho, \lambda_g^z, b)$ of our reference economy are specified: $b = -10$, $\lambda_g^v = 0.0025$, $\lambda_g^\varrho = 0$, $\delta_g^z = -0.30$. Apart from $\lambda_g^\varrho = 0$ all parameter are close to the maximal values feasible under the restriction that $\Omega^j = \Omega$ be positive definite. $\lambda_g^\varrho = 0$ is a simplification. Our convention regarding the definition of optimism implies $b<0$. *Parameter variability in all models below are policy related.*

In the rest of this paper we study fluctuations and the effect of monetary policy by computing equilibria with perturbation methods using a program of Hehui Jin (see Jin and Judd, 2002; Jin, 2003). A solution is declared an equilibrium if: (i) a model is approximated by at least second order derivatives; (ii) errors in market clearing conditions and Euler equations are less than 10^{-3}. Since steady state consumption is about 0.7 the permitted error is 1/500 of this marginal utility.

4. The role of technology, expectation and money shocks in economic fluctuations

4.1. Business cycle fluctuations in the model with money shocks

In Table 1 we compare the volatility of the classical RBC model under REE (without capacity utilization) with the volatility of the RBE with money shocks. It shows that although σ_υ is a fraction of 0.072, the RBE reproduces well the U.S. record. Without friction our model does not perform well in the labor market; without sufficient resource under-employment it does not capture the low volatility of the wage rate, the low correlation between the wage rate and GNP and the high volatility of hours. However, these shortcomings do not diminish its value for the study of monetary policy.

The correlation between consumption and GNP is a central problem for an RBE and reveals the complexity of dynamics when fluctuations are propagated by expectations. In a standard RBC model too high correlation among aggregate variables results from the large persistent technological shocks which increase GNP, investments and consumption together. When expectations of high future returns drive high investment rate, a competitive force emerges between investment and consumption. A date t increased output which is associated with increased agent's expected return on investments leads to increased investment but tends to reduce date t consumption. This force leads to a negative correlation between consumption and GNP. Kurz et al. (2003) show the potential dominance of this factor. An opposite force operates when increased investments together with increased capacity utilization result in higher date $t+1$ output, making increased output and consumption possible, causing positive correlation between them. Such positive correlation is driven by persistence in beliefs which generate the higher investments and capacity utilization to begin with. Persistence of beliefs expressed by $\lambda_z = 0.65$, $\lambda_\upsilon^z = 8.00$ and the condition $\lambda_g^z = -0.30$ *are both needed for the positive* 0.73 *correlation between output and consumption seen in Table* 1.

4.1.1. Decomposing the effect of technology, expectations and money shocks

Fluctuations in the RBE are caused by technology, expectations and monetary shocks. What are the contributions of these three factors? Table 2 provides the answer. σ_X is the standard deviation of X and $\rho(X, Y)$ is the correlation of X with

Table 1
Comparing the volatility of the RBE with the classical RBC[5]

(Percent, all data H-P filtered)

Variable	Standard deviation of variable			Correlation of variable with GNP		
	RBC with $\sigma_\upsilon = 0.0072$	U.S. data	RBE with $\sigma_\upsilon = 0.003$	RBC with $\sigma_\upsilon = 0.0072$	U.S. data	RBE with $\sigma_\upsilon = 0.003$
Y	1.39	1.81	1.82	1.00	1.00	1.00
I	4.09	5.30	5.24	0.99	0.80	0.94
C	0.61	1.35	0.93	0.94	0.88	0.73
L	0.67	1.79	1.02	0.97	0.88	0.87
W	0.75	0.68	1.05	0.98	0.12	0.88
π	na	1.79	2.91	na	0.24	0.33

Table 2
Decomposing the components of business cycles

(Percent, all data H-P filtered)

X	RBE with random money growth		RBE with constant money growth		REE with constant money growth	
	σ_X	$\rho(X, Y)$	σ_X	$\rho(X, Y)$	σ_X	$\rho(X, Y)$
Y	1.82	1.00	1.75	1.00	0.81	1.00
I	5.24	0.94	5.02	0.95	1.97	0.99
C	0.93	0.73	0.89	0.74	0.39	0.99
L	1.02	0.87	0.97	0.88	0.33	0.99
W	1.05	0.88	1.01	0.89	0.48	0.99
π	2.91	0.33	1.60	0.57	0.73	−0.10

GNP. Column 1 reproduces the data in Table 1. Next, we shut off the random money shock by setting $\sigma_\varrho = 0$ hence money grows at a constant rate υ^*. Column 3 reports results for the REE with constant money growth and flexible capacity utilization. If we think of REE volatility as measuring the effect of technology, then we arrive at the following *rough approximation*:

- 40% of real fluctuations in the model are due to technological shocks and capacity utilization;
- 4% of real fluctuations are due to monetary shocks amplified by agents' expectations;

[5]Results for the standard RBC model with $\sigma_\upsilon = 0.0072$ are from King and Rebelo (1999, Table 3). Data for the U.S. economy are from Stock and Watson (1999) except for inflation which is measured by the GNP deflator and which we computed for the entire period 1947:1–2003:2. Stock and Watson (1999) computed the data only for the period 1953:1–1996:4.

- 56% of real fluctuations in the model are *demand driven*, due to pure expectations of agents.

Since prices adjust immediately to money supply changes, the REE is strongly neutral: *unanticipated effects of money are impossible.* Nevertheless, this REE is of interest at it measures volatility caused by technology and is thus a useful reference to evaluate the effect of any monetary policy rule.

4.1.2. Money non-neutrality, Phillips curve and sticky prices in the RBE

Let us now examine two monetary properties of the RBE reported in Tables 1 and 2.

(i) *Phillips curve behavior*. By simulating 10,000 observations of the reference economy we can estimate the following statistical Phillips curve which is compatible with many published estimates:

$$\pi_t - \pi^* = 0.1754[\log(y_t) - \log(y^*)] + 0.4272[\pi_{t-1} - \pi^*] + 0.0227[\pi_{t-2} - \pi^*]. \tag{28}$$

(ii) *Money is non-neutral in an RBE and prices are sticky*. Money non-neutrality and impulse response behavior in an RBE were discussed by Kurz et al. (2003). We now add the fact that agents in the RBE have diverse belief about the effects of money shocks. Hence money shocks may *increase* or *decrease* output and consumption, depending upon the structure of beliefs. $\lambda_{\varrho}^{z} > 0$ is a sufficient condition to ensure that *a positive* money shock causes *a positive* impulse of real variables. This condition requires that positive money shock should lead agents to expect an increased level of market confidence. Since endogenous variables are functions of (z_t^1, z_t^2), forecasts of (z_{t+1}^1, z_{t+1}^2) are *forecasts of endogenous variables*. When agents are confident about the future they increase their consumption and demand for money. Hence, a positive monetary shock which leads to higher forecast of (z_{t+1}^1, z_{t+1}^2) actually leads to an increased demand for money and thus reduces the inflationary impact of the monetary shock. We arrive at the same conclusion by examining the equilibrium inflation function π_t which depends upon ϱ_t. Indeed, $\partial\pi_t/\partial\varrho_t$ is the proportion of a money shock translated into inflation. In an REE, $\partial\pi_t/\partial\varrho_t = 1$ but in our reference RBE $\partial\pi_t/\partial\varrho_t = 0.79$. An econometrician studying the relation between money and inflation in this RBE may conclude prices are sticky since they do not respond fully to money shocks. This conclusion suggests that empirical evaluation of sticky price models requires caution and must identify the cause of price movements which appear to be sticky.

For the rest of this paper *we use the volatility in the first column of Table* 2 *as a reference to measure the efficacy of any monetary policy*. Under a Friedman rule of constant money growth, volatility can be reduced to a level specified in the *second column*. If, in addition, all pure effects of beliefs were neutralized by a central bank policy, fluctuations would be reduced to a level determined by technology as in the *third column* of Table 2. We thus put forward the following two questions:

(A) Is there a policy rule for which the economy attains the same level of volatility that would be attained by a constant growth of money?

(B) Is there a policy rule for which the economy attains the level of volatility that would be attained in an REE and be determined by technology only?

5. Economic stabilization with a monetary rule

5.1. What is the objective of central bank policy?

What is the aim of a central bank when fluctuations are caused by forces outlined in our theory? It is a standard argument in the policy debate that monetary policy should minimize the expected discounted loss of future fluctuations of inflation and GNP growth. This is the standard procedure of deducing the policy rule from a bank's objective. We comment here on two issues which are related to this objective. The first relates to the source of fluctuations and the second to the probability belief which the central bank should adopt.

Kurz et al. (2003) argue (pp. 222–225) that, as in our model economy, market beliefs constitute an externality which generates excess real economic fluctuations. This leads us to two observations. First, economic fluctuations are undesirable and hence society may aim to eliminate them or reduce their impact. Since a significant fraction of economic fluctuations is a man made externality, monetary policy is an effective tool to suppress the effect of market beliefs on fluctuations. In short, they argue that in an RBE a monetary policy rule is a desirable and effective stabilization tool. *This remains our perspective in this paper*.

In an economy with diverse probability beliefs what is the belief that should be adopted by the central bank when conducting the optimization leading to the policy rule? A central bank recognizes that when agents hold diverse beliefs, some are wrong. Since it has the same information as private agents, a bank cannot determine whose beliefs are right. Consequently, a bank's policy must take a symmetric view of belief diversity: over time any agent may hold a correct belief and policy must be optimal with respect to an average belief of agents over time. The RBE rationality conditions imply that the mean belief of any agent over time is exactly the non judgmental stationary empirical distribution. Hence a symmetric perspective would propose that public policy be based on evaluating future cost and benefits using the stationary probabilities of an RBE.

In this paper we avoid the formulation of an explicit utility function of the central bank since we want to focus on a study of the set of *feasible* policies. With this in mind we now turn to the examination of outcomes generated by a Taylor (1993) policy rules of the type

$$\log \frac{1+r_t}{1+r^*} = v_y \log\left(\frac{y_t}{y^*}\right) + v_\pi(\pi_t - \pi^*), \tag{29}$$

where (r^*, y^*) are equal to the steady state values and $\pi^* = 0.01$ hence the economy has a positive long run inflation rate. We then study in Section 5.3 a version of this rule with a random term, interpreted as a discretionary component of the policy, not known to the market in advance.

5.2. Policy rules without discretion

In all policy experiments the real economy is the reference economy. To examine the effect of rule (29) we turn off the money shocks (i.e. set $\varrho_t = 0$) in the reference economy and replace it with (29). Hence, to study the impact of policy, one must focus only on the *difference* between numbers reported in the tables below and the volatility in the reference economy for which

$$\sigma_Y = 1.82, \quad \sigma_I = 5.24, \quad \sigma_C = 0.93, \quad \sigma_\pi = 2.91.$$

We also compare volatility under rule (29) with the same economy with a constant growth of money.

Table 3 reports HP-filtered standard deviations of output, investment, consumption and inflation under the rule (29). Also, the unconditional correlation between inflation and output. It demonstrates that monetary policy can have a major stabilization effect: there are many policies which lead to reduced real volatility, relative to the reference economy, by a wide margin. The table demonstrates the subtle tradeoff between real volatility and inflation volatility which can be attained using different policy instruments. Some specific points to note are as follows:

(i) *Most efficient stabilization policies require joint policy instruments*. Efficient policy rules must consider the joint instruments v_y and v_π in order to avoid increased volatility of *both* inflation and output. Bold borders indicate policy rules which attain, for each v_π, *minimal consumption volatility*.

(ii) *Apart from inflation, outcomes of policy instruments are* ***not*** *monotonic*. If you fix v_π and vary v_y across the table, the volatility of real variables usually declines first and then rises. If you fix $v_y > 0.4$ and vary v_π going down the table you find the volatility of consumption and investment declining firs and then rising. However, the effects of each of these instruments on inflation volatility is monotonic: increasing v_π decreases inflation volatility while increasing v_y increases it.

(iii) $v_\pi = \infty$ *is an efficient policy*. It attains a zero inflation volatility independently of v_y.

(iv) Is there a rule attaining the outcome of a constant money growth for which $\sigma_Y = 1.75$, $\sigma_I = 5.02$, $\sigma_C = 0.89$, $\sigma_\pi = 1.60$? The answer to this question (A) is ***yes***! A rule such as ($v_\pi = 1.3, v_y = 0.1$) attains results which are very close to those attained by a constant money growth policy.

(v) *The strongest tradeoff is between consumption and inflation volatility*. Inflation volatility can be reduced to zero and consumption volatility by as much as 52% relative to the reference economy. Table 3 offers monetary policy a rich tradeoff which is not available in economies with sticky prices. For example, a standard sticky price model of Rotemberg and Woodford (1999) offers no feasible tradeoff as seen in Figs. 2.5 and 2.6 (pp. 85–86). This leads to the conclusion that both output and inflation can be stabilized with $v_\pi = \infty$, unless the central bank has a more complex objective.

(vi) *There are rules that dominate the constant money growth rule*. Assuming society prefers less volatility we see that there are rules which dominate the constant money growth policy for any utility function (e.g. $v_\pi = 10, v_y = 0.9$). We also show in

Table 3
Efficacy of alternative monetary rules: no discretion

(Percent standard deviations or correlation, all data H-P filtered)

v_π	$v_y \rightarrow$	0	0.1	0.2	0.3	0.4	0.5	0.7	0.9	1.0	5.0
1.1	Y	1.80	1.70	1.61	1.54	1.48	1.44	1.43	1.48		
	I	5.26	4.79	4.51	4.48	4.68	5.08	6.28	7.77		
	C	0.98	0.79	0.62	0.51	0.47	0.53	0.83	1.21	Unstable	Unstable
	π	0.35	4.82	9.94	15.00	20.02	24.99	34.79	44.44		
	$\rho(\pi, Y)$	0.43	−0.57	−0.63	−0.69	−0.76	−0.82	−0.92	−0.97		
1.2	Y	1.80	1.74	1.68	1.63	1.59	1.55	1.49	1.44	1.43	
	I	5.26	4.97	4.73	4.57	4.48	4.47	4.65	5.06	5.34	
	C	0.98	0.87	0.76	0.67	0.59	0.52	0.47	0.52	0.58	Unstable
	π	0.26	2.66	5.55	8.41	11.24	14.05	19.59	25.04	27.74	
	$\rho(\pi, Y)$	0.42	−0.6	−0.63	−0.66	−0.69	−0.72	−0.78	−0.84	−0.87	
1.3	Y	1.79	1.75	1.71	1.67	1.64	1.61	1.55	1.50	1.49	
	I	5.26	5.04	4.85	4.70	4.59	4.51	4.47	4.56	4.65	
	C	0.98	0.90	0.82	0.75	0.68	0.62	0.53	0.47	0.47	Unstable
	π	0.20	1.84	3.86	5.87	7.85	9.81	13.69	17.52	19.41	
	$\rho(\pi, Y)$	0.41	−0.64	−0.65	−0.67	−0.69	−0.71	−0.75	−0.79	−0.80	
1.4	Y	1.79	1.76	1.73	1.70	1.67	1.64	1.59	1.55	1.53	
	I	5.26	5.08	4.92	4.79	4.67	4.59	4.48	4.47	4.49	
	C	0.98	0.91	0.85	0.79	0.73	0.68	0.59	0.52	0.50	Unstable
	π	0.17	1.41	2.97	4.51	6.04	7.56	10.55	13.50	14.96	
	$\rho(\pi, Y)$	0.4	−0.68	−0.68	−0.69	−0.71	−0.72	−0.75	−0.77	−0.79	
1.5	Y	1.79	1.76	1.74	1.71	1.68	1.66	1.62	1.58	1.56	
	I	5.26	5.10	4.96	4.84	4.74	4.65	4.53	4.47	4.46	
	C	0.98	0.92	0.87	0.81	0.77	0.72	0.64	0.57	0.54	Unstable
	π	0.14	1.15	2.42	3.67	4.92	6.16	8.60	11.00	12.19	
	$\rho(\pi, Y)$	0.39	−0.71	−0.71	−0.72	−0.73	−0.74	−0.76	−0.78	−0.79	
10	Y	1.80	1.79	1.79	1.78	1.77	1.77	1.76	1.75	1.74	1.60
	I	5.57	5.24	5.21	5.18	5.15	5.12	5.06	5.01	4.98	4.49
	C	0.98	0.97	0.96	0.95	0.94	0.93	0.90	0.88	0.87	0.60
	π	0.02	0.07	0.15	0.23	0.30	0.38	0.53	0.68	0.75	3.54
	$\rho(\pi, Y)$	0.32	−0.99	−0.99	−0.99	−0.99	−0.99	−0.99	−0.99	−0.99	−0.99
∞	Y	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
	I	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28
	C	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
	π	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\rho(\pi, Y)$	na	na	na	na	na	na	na	na	na	na

Table 4
Estimated Phillips curves for monetary rules with $v_\pi = 1.4$; no discretion

Model	Variable	$v_y = 0$	$v_y = 0.1$	$v_y = 0.2$	$v_y = 0.3$	$v_y = 0.4$	$v_y = 0.5$	$v_y = 0.7$	$v_y = 0.9$	$v_y = 1.0$	$v_y = 5.0$
OLS	const.	0.009	−0.129	−0.245	−0.232	−0.053	−0.350	2.041	5.026	6.894	20.269
	Log(y)	−0.015	0.258	0.504	0.485	0.116	−0.720	−4.260	−10.531	−14.459	−42.819
	π_{t-1}	0.907	1.583	1.525	1.380	1.189	0.951	0.338	−0.378	−0.721	0.029
	π_{t-2}	0.044	−0.396	−0.351	−0.300	−0.244	−0.181	−0.035	0.116	0.181	0.037
INST	const.	0.009	−0.110	−0.279	−0.321	−0.174	−0.220	2.005	5.230	7.233	20.265
	Log(y)	−0.015	0.221	0.574	0.667	0.368	−0.451	−4.185	−10.959	−15.172	−42.811
	π_{t-1}	0.907	1.528	1.573	1.462	1.273	1.022	0.351	−0.439	−0.811	0.029
	π_{t-2}	0.044	−0.379	−0.364	−0.321	−0.264	−0.198	−0.038	0.128	0.199	0.037

Table 7 improved policy rules with inertia that dominate the constant money growth policy.

(vii) A stabilization activist policy (i.e. $v_y > 0$) which is too aggressive *can destabilize the economy*.

(viii) *There is no policy rule under* (29) *that attains the level determined by technology only*, *which is* $\sigma_Y = 0.81$, $\sigma_I = 1.97$, $\sigma_C = 0.39$, $\sigma_\pi = 0.73$. Hence, the answer to question (B) in Section 4 is ***no***, the simple rule (29) cannot jointly stabilize to the level determined by technology only.

(ix) *The results do not depend upon the reference economy*. Without exhibiting additional equilibria we observe the qualitative results listed here continue to hold for all feasible belief parameter values.

5.2.1. Activist monetary policy 'destroys' the statistical Phillips curve

Policy choices are expressed in our model with the two instruments (v_π, v_y). It is thus not what are the policy choices that are suggested by a Phillips curve which is estimated from the equilibrium data of an economy *with given policy instruments*. In Table 3 we report unconditional correlations between inflation and Log(GNP) and it is clear that as the policy becomes more activist (i.e. $v_y > 0$), this correlation tends to −1. Focusing on conditional correlations, we report in Table 4 estimates of the same statistical Phillips curve as in (28). We study two specifications of the model: one is OLS and the second (INST) uses instrumental variables where all exogenous and lagged exogenous variables are used as instruments, given a policy instrument $v_\pi = 1.4$ and for rising values of v_y. Each estimate is made from data generated by an equilibrium with the specified policy rule.

There are three clear conclusions which emerge from Table 4:

(i) The statistical Phillips curve is a relationship between two endogenous variables and is extremely sensitive to the policy regime which prevails in that equilibrium.
(ii) For moderately activist policy rules with $v_y \leqslant 0.4$ the statistical Phillips curve estimated for each equilibrium is similar to the estimates which are obtained from U.S. data.
(iii) *As policy becomes more activist the statistical Phillips curve becomes essentially vertical*. Since the curve changes dramatically with the policy, there is no sense in

which it reflects policy choices along the curve; there are no policy instruments to accomplish such choices.

These results are compatible with existing ideas in monetary economics and question the use of a fixed Phillips curve in a policy model. Some authors estimate from data a Phillips curve, *treat it as a fixed structural equation* and perform policy experiments, assuming the economy moves along that curve. Our General Equilibrium approach suggests this procedure is flawed.

5.3. Outcomes under policy rules with discretion

We now introduce a discretionary component to the monetary rule. It is formulated as a modified rule containing a random component d_t so that the rule takes the form

$$\log \frac{1+r_t}{1+r^*} = v_y \log\left(\frac{y_t}{y^*}\right) + v_\pi(\pi_t - \pi^*) + d_t \tag{30a}$$

with an empirical distribution

$$d_{t+1} = \lambda_d d_t + \rho^d_{t+1} \quad \text{where } \rho^d_{t+1} \sim \mathrm{N}(0, \sigma_d). \tag{30b}$$

Rule (30b) does not mean the central bank uses a randomized strategy. Rather, d_t is ex-ante unknown to the markets and reflects the public's uncertainty about the future rule. 'Discretion' d_t also includes the bank's unknown reaction to unexpected shocks such as an oil shock, etc. Agents know (30a)–(30b) but disagree at each t about the distribution of d_{t+1}. This opens the door for market's beliefs about d_{t+1} to impact the efficacy of the policy. That is, when the policy contains a discretionary variable d_t, the market views as a 'surprise,' diverse beliefs about future surprises are rationalizable. We show that policy outcomes are altered by such effects, offering risks and opportunities to a bank's policy.

The introduction of d_t generates a new policy model but it is mathematically the same as the model in (17) and (21a). Formally d_t replaces ϱ_t, but with drastically different model implications. Without rewriting (17) and (21a) observe the parameters $(\lambda_\varrho, \sigma_\varrho, \lambda^\varrho_g, \lambda^z_\varrho)$ are replaced by a new vector $(\lambda_d, \sigma_d, \lambda^d_g, \lambda^z_d)$. Consider first (λ_d, σ_d) . McCallum and Nelson (1999) study a rule with a random term and estimate $\sigma_d = 0.0017$ but allow lagged variables and inertia in the rule. We assume $\pi^* = 1\%$ per quarter hence we postulate $\sigma_d = 0.0025$ in order to study a discretion with standard deviation of 25 basis points. Rudebusch (2002) argues policy inertia is inconsistent with lack of predictability of changes in short rate and suggests that measured inertia arises from persistence in discretion. This is expressed by $\lambda_d > 0$ and we study the hypothetical case of $\lambda_d = 0.50$. Variations in these parameters have small effects and do not alter our *qualitative* conclusions. The parameters $(\lambda^d_g, \lambda^z_d)$ describe beliefs about discretion and before discussing the issues they raise, we first interpret them:

λ^d_g – effect of an agent belief state on his $t+1$ forecasted central bank discretionary decisions.

λ_d^z – the effect of a bank's discretionary surprise on the agent's forecasted market state of belief.

It is obvious discretion introduces a random shock into the market and this, *by itself*, *increases volatility*. The main issue is how the added volatility interacts with market beliefs since the presence of a random discretionary component triggers diverse beliefs, at any date t, about *abnormal future interest rates*. The range of values which $(\lambda_g^d, \lambda_d^z)$ can feasibly take describes those rationalizable beliefs which may exist in the market when a discretionary element is present in the policy rule.

Now, the parameter λ_g^d reflects how an agent's state of belief about future high productivity affects his belief about the bank's discretionary decisions. If $\lambda_g^d > 0$ an agent who believes in abnormally high future productivity growth also believes the central bank is likely to induce a positive discretionary *increase* of rates above the mean in (30a). If $\lambda_g^d < 0$ the opposite is true: an agent who is bullish about high future productivity also believes the central bank is likely to induce an abnormal discretionary *decrease* of rates to accommodate liquidity needs of an economy with high investment rate. *It turns out that both cases are rationalizable* under the RBE rationality principle, but within a very narrow feasible range of $-0.0008 < \lambda_g^d < 0.0008$. In the policy experiments below we study the impact of such market beliefs on the volatility of the economy. We also discuss the desirability of abandoning discretion altogether in favor of fully transparent policy rules.

What about the second parameter λ_d^z? Since d_t is an unexpected shock, it turns out that the parameter λ_d^z has a small effect on volatility which is also not systematic. Hence we ignore it by setting $\lambda_d^z = 0$. We thus study below two hypothetical economies with very modest parameter values of

$$\text{Economy I}: \ \lambda_d^z = 0.00, \quad \lambda_g^d = -0.0006,$$

$$\text{Economy II}: \ \lambda_d^z = 0.00, \quad \lambda_g^d = 0.0006.$$

Why could the discussed effects of market expectations be useful and what is the empirical evidence in favor of Economy type I vs. type II? To explain these issues we start with a simple general principle which can be stated as follows:

Policy principle: The efficacy of a monetary policy increases if public expectations are compatible and supportive of the policy goals. With such additional wind in the policy's sails, the same policy goals can be attained with less aggressive instruments relative to an economy in which market expectations go against the policy and render it less effective.

To illustrate how this principle works consider a state when, due to their optimism, agents increase planned consumption and investments conditional on the policy in place. Agents can forecast the bank's *normal* interest rate using (30a) but what about the bank's discretion? If they expect bank's discretion to accommodate the liquidity needs of an expected abnormal burst of productivity and investments, they would expect the bank to lower the nominal rate (i.e. $d_{t+1} < 0$). This means $\lambda_g^d < 0$: when optimistic, agents expect the central bank to accommodate the implied liquidity needs of an abnormally high growth and high investment boom. Now consider the opposite. If agents expect bank's discretion to resist abnormal bursts of productivity and investments, they would expect the bank to raise the nominal rate (i.e. $d_{t+1} > 0$). This means $\lambda_g^d > 0$: when optimistic, agents expect the central bank to resist the abnormal growth and investments by raising rates.

We observe that both patterns of behavior are rationalizable. However, we show below that for all policy instruments *Economy I with* $\lambda_g^d < 0$ *is more volatile than Economy II* with $\lambda_g^d > 0$. In terms of the above policy principle, this means that when $\lambda_g^d < 0$ private expectations in Economy I operate against the policy goals resulting in reduced efficacy of policy in Economy I relative to Economy II. Similarly, in Economy II private expectations are aligned with the policy goals hence they bolster and support them. Since behavior under $\lambda_g^d > 0$ bolsters policy, the central bank prefers private expectations to have the structure in Economy II. However, the bank does not choose the pattern of market beliefs; the bank's only choice is whether to use discretion and this choice should be influenced by the empirical evidence regarding the structure of private expectations. We discuss this choice later, after examining the simulation results.

5.3.1. Policy rules with discretion: Economy I

In Table 5 we report results of policy experiments for Economy I. The results show that a central bank's decision whether to use discretion or employ full transparency in the policy rule has effects on market volatility. There are three clear conclusions which emerge from Table 5:

(i) For non activist policies with $v_y = 0$ central bank's discretion causes a *dramatic* rise in the volatility of inflation due to the random discretionary element which is not present in Table 3. This rise in volatility is caused by the bank's discretionary decision making process.
(ii) For all policy rules, central bank discretion increases the volatility of real variables relative to results in Table 3. For $v_y > 0.1$ the rise is not dramatic but could add some 5% to the volatility of consumption. Hence, the belief structure in Economy I *works against the policy goals*:
(iii) The effect of bank's discretion on volatility falls sharply as stabilization policy becomes more aggressive in terms of higher values of v_y.

We make two additional observations:

(i) The pattern of nonmonotonic effects of policy instruments and the effect of policy instruments on the estimated Phillips curves are the same in Table 5 as in Table 3 and are not be repeated here.
(ii) The very strong policy $v_\pi = \infty$ results in exactly the same volatility in all three cases of Table 3, Table 5 and Table 6 showing that aggressive anti inflationary policy can neutralize all effects of central bank discretion. We also note that under $v_\pi = \infty$ the level of volatility is virtually the same as in the reference RBE economy except that the volatility of inflation is reduced to 0.

Table 5
Volatility increasing discretion

(Percent standard deviations or correlation, all data H-P filtered)

v_π	$v_y \rightarrow$	0	0.1	0.2	0.3	0.4	0.5	0.7	0.9	1.0	5.0
1.1	Y	1.83	1.72	1.63	1.56	1.50	1.45	1.42	1.47	1.52	
	I	5.44	4.91	4.57	4.46	4.59	4.94	6.07	7.53	8.31	
	C	1.04	0.84	0.67	0.54	0.47	0.50	0.77	1.15	1.35	Unstable
	π	2.13	4.36	9.28	14.29	19.29	24.25	34.05	43.70	48.48	
	$\rho(\pi, Y)$	0.19	−0.54	−0.61	−0.68	−0.74	−0.80	−0.90	−0.96	−0.98	
1.2	Y	1.83	1.77	1.71	1.66	1.61	1.57	1.50	1.45	1.44	
	I	5.44	5.11	4.85	4.65	4.52	4.47	4.57	4.93	5.18	
	C	1.04	0.92	0.82	0.72	0.63	0.56	0.47	0.50	0.55	Unstable
	π	1.78	2.49	5.10	7.88	10.68	13.47	19.00	24.46	27.15	
	$\rho(\pi, Y)$	0.18	−0.53	−0.61	−0.65	−0.68	−0.71	−0.77	−0.82	−0.85	
1.3	Y	1.83	1.78	1.74	1.70	1.67	1.63	1.57	1.52	1.50	
	I	5.43	5.19	4.98	4.81	4.67	4.56	4.47	4.51	4.58	
	C	1.04	0.95	0.87	0.80	0.73	0.66	0.56	0.49	0.47	Unstable
	π	1.53	1.83	3.55	5.47	7.41	9.36	13.22	17.04	18.93	
	$\rho(\pi, Y)$	0.17	−0.53	−0.64	−0.66	−0.68	−0.70	−0.74	−0.77	−0.79	
1.4	Y	1.83	1.79	1.76	1.72	1.69	1.66	1.61	1.57	1.55	
	I	5.43	5.23	5.06	4.90	4.77	4.66	4.52	4.47	4.47	
	C	1.03	0.97	0.90	0.84	0.78	0.73	0.63	0.55	0.52	Unstable
	π	1.34	1.49	2.75	4.21	5.70	7.19	10.17	13.11	14.57	
	$\rho(\pi, Y)$	0.16	−0.53	−0.66	−0.69	−0.71	−0.71	−0.74	−0.77	−0.78	
1.5	Y	1.83	1.79	1.76	1.74	1.71	1.68	1.64	1.60	1.58	
	I	5.43	5.25	5.10	4.96	4.84	4.74	4.58	4.49	4.47	
	C	1.03	0.97	0.92	0.87	0.82	0.77	0.68	0.60	0.57	Unstable
	π	1.19	1.28	2.26	3.43	4.64	5.85	8.28	10.68	11.87	
	$\rho(\pi, Y)$	0.16	−0.52	−0.68	−0.71	−0.72	−0.73	−0.75	−0.77	−0.78	
10	Y	1.80	1.80	1.79	1.79	1.78	1.77	1.76	1.75	1.75	1.60
	I	5.31	5.28	5.25	5.22	5.19	5.15	5.10	5.04	5.02	4.50
	C	1.00	0.98	0.97	0.96	0.95	0.94	0.92	0.90	0.89	0.61
	π	0.11	0.12	0.18	0.24	0.31	0.39	0.53	0.68	0.75	3.54
	$\rho(\pi, Y)$	0.11	−0.52	−0.80	−0.90	−0.94	−0.96	−0.97	−0.98	−0.98	−0.99
∞	Y	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
	I	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28
	C	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
	π	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\rho(\pi, Y)$	na	na	na	na	na	na	na	na	na	na

Table 6
Volatility decreasing discretion

(Percent standard deviations or correlation, all data H-P filtered)

v_π	$v_y \rightarrow$	0	0.1	0.2	0.3	0.4	0.5	0.7	0.9	1.0	5.0
1.1.	Y	1.76	1.67	1.59	1.52	1.47	1.44	1.43	1.50	1.56	
	I	5.10	4.68	4.48	4.51	4.78	5.23	6.50	8.01	8.81	
	C	0.92	0.74	0.58	0.48	0.48	0.56	0.88	1.27	1.47	Unstable
	π	1.75	5.80	10.79	15.80	20.78	25.73	35.51	45.14	49.90	
	$\rho(\pi, Y)$	−0.00	−0.53	−0.62	−0.70	−0.77	−0.83	−0.93	−0.97	−0.98	
1.2	Y	1.76	1.71	1.66	1.61	1.57	1.53	1.47	1.44	1.43	
	I	5.10	4.83	4.64	4.51	4.47	4.49	4.74	5.21	5.50	
	C	0.92	0.81	0.71	0.62	0.55	0.50	0.47	0.56	0.62	Unstable
	π	1.50	3.55	6.29	9.09	11.89	14.67	20.18	25.61	28.30	
	$\rho(\pi, Y)$	−0.01	−0.51	−0.60	−0.64	−0.69	−0.72	−0.79	−0.85	−0.87	
1.3	Y	1.76	1.72	1.68	1.65	1.62	1.59	1.53	1.49	1.47	
	I	5.10	4.90	4.74	4.62	4.53	4.48	4.49	4.63	4.74	
	C	0.92	0.84	0.77	0.70	0.64	0.58	0.50	0.47	0.47	Unstable
	π	1.31	2.64	4.51	6.45	8.40	10.34	14.19	17.99	19.87	
	$\rho(\pi, Y)$	−0.015	−0.5	−0.60	−0.64	−0.68	−0.70	−0.75	−0.79	−0.81	
1.4	Y	1.76	1.73	1.70	1.67	1.64	1.62	1.57	1.53	1.51	
	I	5.10	4.94	4.80	4.68	4.60	4.53	4.47	4.49	4.53	
	C	0.92	0.86	0.80	0.74	0.69	0.64	0.56	0.50	0.48	Unstable
	π	1.16	2.13	3.54	5.02	6.52	8.01	10.98	13.91	15.36	
	$\rho(\pi, Y)$	−0.02	−0.5	−0.61	−0.66	−0.68	−0.71	−0.74	−0.78	−0.79	
1.5	Y	1.77	1.74	1.71	1.68	1.66	1.64	1.59	1.56	1.54	
	I	5.10	4.96	4.84	4.73	4.65	4.58	4.49	4.46	4.47	
	C	0.92	0.86	0.81	0.76	0.72	0.68	0.60	0.54	0.51	Unstable
	π	1.04	1.79	2.92	4.12	5.33	6.55	8.96	11.35	12.53	
	$\rho(\pi, Y)$	−0.02	−0.51	−0.62	−0.67	−0.7	−0.72	−0.75	−0.78	−0.79	
10	Y	1.79	1.78	1.78	1.77	1.77	1.76	1.75	1.74	1.73	1.59
	I	5.23	5.20	5.17	5.14	5.10	5.08	5.02	4.97	4.95	4.48
	C	0.97	0.95	0.94	0.93	0.92	0.91	0.89	0.87	0.86	0.59
	π	0.11	0.13	0.18	0.25	0.32	0.39	0.54	0.69	0.76	3.54
	$\rho(\pi, Y)$	−0.06	−0.55	−0.81	−0.89	−0.93	−0.95	−0.97	−0.97	−0.98	−0.99
∞	Y	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
	I	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28
	C	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
	π	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00
	$\rho(\pi, Y)$	na	na	na	na	na	na	na	na	na	na

5.3.2. Policy rules with discretion: Economy II

Table 6 reports results of policy experiments with a specifications of Economy II. Clearly, all volatility measures in Table 6 are lower than in Table 5, hence the structure of beliefs in Economy II works in support of the policy goals. We conclude that if the central bank must use discretion, then the belief structure of Economy II bolsters the policy while the belief structure in Economy I diminishes the efficacy of policy. Table 6 reveals results which are analogous to those reported for Table 5: (i) big rise in inflation volatility for non activist policies with $v_y = 0$; (ii) discretion, as such, increases volatility while the belief structure ($\lambda_g^d > 0$) reduces it, implying lower real volatility for most efficient policies relative to the non discretionary case in Table 3; (iii) aggressive stabilization instruments can suppress the negative impact of bank's discretion. Also, the nonmonotonic effect of policy instruments and the effect of policy instruments on the estimated Phillips curves in Table 6 are the same as in Table 3.

The beliefs patterns in Economies I and II are both rationalizable, but which of the two better reflects the empirical record? We do not have conclusive evidence but the recent experience of the 1996–2001 period provides a hint. During that time, the economy experienced high, above normal, growth rate and most private observers demanded and predicted that the central bank follow a low discretionary interest rate policy, accommodating the liquidity needs of an abnormally high growth economy. This implies an expectations parameter $\lambda_g^d < 0$ which is the norm of Economy I. Hence, the qualitative results of Table 5 are likely to be the correct ones, rather than $\lambda_g^d > 0$ as in Table 6.

We add that during the 1996–2003 episode the Fed executed a discretionary policy of low interest rate which contributed to the intense investment boom of 1996–1999. Claiming to detect a structural break of a high and persistent productivity growth regime, the Fed maintained very low interest rates up to very late in June of 1999 when it began to raise rates. Late in 2000 it became clear that the over-heated investment boom was weakening and in January 2001 the Fed drastically reversed course starting a historical reduction of the fund's rate during 2001. Evidence in support of the Fed's claim of a structural break into high productivity regime is limited. Our theory proposes that had the Fed followed a non discretionary policy it would have raised rates much earlier, slowing early the economy's excess. The resulting level of real and financial volatility in 1996–2003 would have been lower. The Fed's discretionary 'judgment' contributed to the actual volatility during this time.

Should discretion be practices by a central bank? There are unique circumstances like a war or an imminent collapse of a major financial institution when discretion has obvious social benefits not in our model. However, recognizing that discretion is costly, our theory leads to several conclusions:

(i) If a bank follows a mild, non activist anti inflation policy such as ($v_\pi \leqslant 1.5, v_y = 0$) then the effects of discretion on inflation volatility are large and in this case a central bank should *abandon discretion altogether.*

(ii) If a bank follows a strong stabilization policy such as ($v_\pi > 1.5, v_y > 0.3$) then the effects of discretion are small and in practice may be disregarded. *Discretion has very low cost*.

(iii) In most intermediate cases discretion has significant cost which depend upon the structure of market expectations. Discretion may be a desirable feature of the policy rule if the belief structure in accord with Economy II. The limited evidence does not support this case.

Notwithstanding the above, we note that a central bank does not have better information or superior ability to make economic judgments than the private sector. Hence, the real social gain from discretion can arise only in those unequivocal catastrophic circumstances when there is no objection to a central bank's discretion. As a result, the weight of the argument supports the conclusion that central bank policy *should be transparent and should abandon discretion except for most unusual circumstances*.

5.4. On inertia and inflation volatility

All policy rules studied up to now implied extremely high equilibrium volatility of inflation. While recorded inflation volatility is around 2% our models predict much higher, counterfactual, levels of inflation volatility induced by any activist policy rule. This problem is resolved by the introduction of inertia into the policy rule. We thus consider the rule

$$\log\frac{1+r_t}{1+r^*} = v_y \log\left(\frac{y_t}{y^*}\right) + v_\pi(\pi_t - \pi^*) + \alpha\left[\log\frac{1+r_{t-1}}{1+r^*}\right]. \tag{31}$$

The available econometric estimates for α are around 0.8 and this is the value we use. We have then computed the RBE under the general policy rule (31) with inertia but without discretion.

In Table 7 we report simulation results for the same policy instruments v_π and v_y as in Table 3. It is clear from the table that with inertia the volatility level of inflation is drastically reduced and for most efficient policy rules it is less than 2.5% per quarter, which is compatible with the empirical record. The reason for this is well known: inertia of 0.8 amplifies the effect of the other instruments by a factor of $\frac{1}{0.2}$ which one can deduce from iterating the policy rule. Table 7, compared with Table 3, reveals some additional and very interesting results.

(i) *Feasible stabilization of real variables is reduced under inertia*. For activist rules with $v_y \leqslant 1$ the lowest consumption volatility is around 0.51% compared to 0.47% without inertia. For output this minimal volatility is 1.43% without inertia and 1.54% with inertia. Under the drastic rule $v_\pi = \infty$ all volatility measures are the same with or without inertia.

(ii) *The rule $v_\pi = 1.4$ and $v_y = 0.3$ dominates the constant money rule*. With ($v_\pi = 1.4, v_y = 0.3$) *we have* $\sigma_C = 0.82$, $\sigma_\pi = 1.54$ *while with Friedman's rule* $\sigma_C = 0.89$, $\sigma_\pi = 1.60$.

Table 7
Monetary rules with inertia: $\alpha = 0.80$, No discretion

(Percent standard deviations or correlation, all data H-P filtered)

v_π	$v_y \rightarrow$	0	0.1	0.2	0.3	0.4	0.5	0.7	0.9	1.0	5.0
1.1	Y	1.79	1.76	1.72	1.69	1.66	1.64	1.59	1.56	1.54	
	I	5.26	5.07	4.91	4.77	4.67	4.59	4.49	4.46	4.47	
	C	0.98	0.91	0.84	0.78	0.73	0.68	0.60	0.53	0.51	Unstable
	π	0.08	0.71	1.47	2.23	3.00	3.77	5.33	6.91	7.70	
	$\rho(\pi, Y)$	−0.44	−0.49	−0.46	−0.47	−0.47	−0.49	−0.51	−0.54	−0.55	
1.2	Y	1.79	1.76	1.73	1.70	1.67	1.65	1.61	1.57	1.56	
	I	5.26	5.08	4.93	4.80	4.70	4.62	4.51	4.47	4.46	
	C	0.98	0.91	0.85	0.80	0.75	0.70	0.62	0.56	0.53	Unstable
	π	0.08	0.62	1.28	1.94	2.61	3.28	4.64	6.01	6.70	
	$\rho(\pi, Y)$	−0.44	−0.55	−0.52	−0.52	−0.52	−0.53	−0.55	−0.57	−0.58	
1.3	Y	1.79	1.76	1.73	1.71	1.68	1.66	1.62	1.58	1.57	
	I	5.26	5.09	4.95	4.83	4.73	4.65	4.53	4.48	4.47	
	C	0.98	0.92	0.86	0.81	0.76	0.72	0.64	0.58	0.55	Unstable
	π	0.07	0.55	1.13	1.72	2.31	2.91	4.10	5.31	5.91	
	$\rho(\pi, Y)$	−0.45	−0.6	−0.57	−0.56	−0.56	−0.57	−0.58	−0.6	−0.61	
1.4	Y	1.79	1.77	1.74	1.71	1.69	1.67	1.63	1.59	1.58	
	I	5.26	5.10	4.97	4.86	4.76	4.68	4.55	4.49	4.47	
	C	0.98	0.92	0.87	0.82	0.78	0.73	0.66	0.60	0.57	Unstable
	π	0.07	0.49	1.02	1.54	2.07	2.61	3.68	4.75	5.30	
	$\rho(\pi, Y)$	−0.45	−0.65	−0.61	−0.6	−0.6	−0.61	−0.62	−0.63	−0.64	
1.5	Y	1.80	1.77	1.74	1.72	1.69	1.67	1.64	1.60	1.59	
	I	5.26	5.11	4.99	4.88	4.78	4.70	4.58	4.51	4.48	
	C	0.98	0.92	0.88	0.83	0.79	0.75	0.68	0.62	0.59	Unstable
	π	0.06	0.45	0.93	1.40	1.88	2.36	3.33	4.31	4.80	
	$\rho(\pi, Y)$	−0.45	−0.69	−0.65	−0.64	−0.64	−0.64	−0.65	−0.66	−0.67	
10	Y	1.80	1.79	1.79	1.78	1.77	1.77	1.76	1.75	1.74	1.64
	I	5.27	5.24	5.21	5.17	5.15	5.11	5.06	5.01	4.98	4.58
	C	0.98	0.97	0.96	0.95	0.94	0.93	0.90	0.88	0.87	0.68
	π	0.01	0.07	0.14	0.20	0.27	0.33	0.46	0.59	0.66	3.33
	$\rho(\pi, Y)$	−0.47	−0.99	−0.99	−0.99	−0.99	−0.99	−0.99	−0.99	−0.99	−0.65
∞	Y	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
	I	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28
	C	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.93	0.98
	π	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
	$\rho(\pi, Y)$	na	na	na	na	na	na	na	na	na	na

(iii) Policy rules with inertia generate negative unconditional correlation between inflation and output. In addition, for activist policy rules in Table 7 our analysis shows that statistical Phillips curves are not present in the data. For activist policies with $v_y > 0.3$ they are essentially vertical.

5.5. *How activist is the current U.S. policy?*

To answer this question we review two key results which bear on the question at hand.

(i) The reference economy with money shocks replicates well the empirical record, including a statistical Phillips curve which matches those estimated with U.S. data.

(ii) All economies under a monetary rule can replicate the real empirical record. However, such economies generate data with negative correlation between output and inflation and in which the statistical Phillips curves are inverted for all activist policy rules with $v_y \geqslant 0.4$. But then all activist monetary rules offer more real stabilization than observed in the real data.

Since real data exhibit a statistical Phillips curve, the policy rules which are compatible with this require $v_y \leqslant 0.4$. But this seems to contradict the econometric estimates which propose the policy rule used in the U.S. is around $v_\pi = 1.5$ and $v_y = 0.5$ with an inertia parameter around 0.8. To explore this issue note that such a policy rule, with or without inertia, implies real volatility which is much lower (i.e. $\sigma_Y = 1.67$, $\sigma_I = 4.70$, $\sigma_C = 0.75$) than the volatility of the reference economy or the volatility observed in the U.S. data. Also, such a rule requires the data to exhibit negative correlation between inflation and output. In all models we examined, ($v_\pi = 1.5, v_y = 0.5$) is not compatible with the estimated statistical Phillips curves, implying incorrect slopes. The only conclusion which is not contradicted by the data is that the policy which the markets believes to be in effect is less activist. For example, to generate an inflation volatility of about 2% the policy $v_\pi = 1.3$ and $v_y = 0.1$ without discretion or inertia generate data which approximate the empirical record, including statistical Phillips curve. With discretion the policy $v_\pi = 1.3$ and $v_y = 0.1$ also approximates the empirical record. With inertia the policy $v_\pi = 1.3$ and $v_y = 0.3$ is a reasonable approximation. Since the Fed has not committed to any particular policy rule, the empirical record would also be affected by the possibility that other variables enter into the policy rules employed by the Fed. This is not explored here.

6. On iterated expectations of market beliefs: why monetary policy has real effects

In Section 1 we explained the money non-neutrality of economies with diverse beliefs. Here we present a second argument to provide a deeper meaning to the effect of diverse beliefs on the efficacy of a monetary policy rule. To do that let us now assume that the number of agents N is large and we denote by $\bar{\mathrm{E}}$ the average market expectations operator, defined by

$$\bar{\mathrm{E}}(\chi) = \frac{1}{N}\sum_{j=1}^{N} \mathrm{E}^j(\chi|g^j) \quad \chi \text{ is a random variable.}$$

Agents condition on their own state of belief g^j hence average market belief entails adding up probabilities which are conditional upon diverse variables. Hence the law of iterated expectations does not hold with respect to the linear operator $\bar{\mathrm{E}}$. To see this precisely note that by definition

$$\begin{aligned}\bar{\mathrm{E}}_t\bar{\mathrm{E}}_{t+1}(\chi_{t+2}) &= \frac{1}{N}\sum_{k=1}^{N} \mathrm{E}^k\left[\left\{\frac{1}{N}\sum_{j=1}^{N} \mathrm{E}^j(\chi_{t+2}|g^j_{t+1})\right\}\middle| g^k_t\right] \\ &= \frac{1}{N}\sum_{k=1}^{N}\frac{1}{N}\sum_{j=1}^{N} \mathrm{E}^k([\mathrm{E}^j(\chi_{t+2}|g^j_{t+1})]|g^k_t).\end{aligned}$$

The law of iterated expectations holds with respect to the belief of any specific agent k hence

$$\mathrm{E}^k([\mathrm{E}^k(\chi_{t+2}|g^k_{t+1})]|g^k_t) = \mathrm{E}^k(\chi_{t+2}|g^k_t).$$

But the belief states and the underlying probabilities Q^j and Q^k are not the same for $k \neq j$, hence there is no sense in which we can arrive at the needed result which is something like

$$\mathrm{E}^k([\mathrm{E}^j(\chi_{t+2}|g^k_{t+1})]|g^k_t) = \mathrm{E}(\chi_{t+2}|g^k_t).$$

Hence, the law of iterated expectations is violated by the market belief operator

$$\bar{\mathrm{E}}_t\bar{\mathrm{E}}_{t+1}(\chi_{t+2}) \neq \bar{\mathrm{E}}_t(\chi_{t+2}).$$

This violation of the law of iterated expectations by market beliefs is generic to an RBE and is at the heart of the high volatility in Kurz (1996, 1997a,b), Kurz and Beltratti (1997), Kurz and Motolese (2001), and Kurz et al. (2003, 2005).

We now return to the Euler equations (6b) and (10b). To simplify we assume capacity utilization is fixed at $\varphi = \varphi^*$ and $v^* = 1$. To log linearize (6b) and (10b) we define

$$\hat{c}^j_t = \log(c^j_t) - \log(c^{j^*}), \quad \hat{\ell}^t_j = \log(\hat{\ell}^j_t) - \log(\ell^{j^*}), \quad \hat{\pi}_t = \pi_t - \pi^*,$$

$$\hat{R}^j_t = R^j_t - R^{j^*}, \quad \hat{r}_t = r_t - r^*$$

and the log linearized equation (6b) is then

$$\gamma\hat{c}^j_t - \zeta(1-\gamma)\hat{\ell}^j_t + \hat{r}_t = \mathrm{E}^j_t[\gamma\hat{c}^j_{t+1} - \zeta(1-\gamma)\hat{\ell}^j_{t+1} + \hat{\pi}_{t+1}]. \tag{31a}$$

Recall that from (10c) we can define $\mathrm{E}^j_t[\beta y^j_{k_{t+1}}] = \mathrm{E}^j_t[R^j_{t+1}]$ to be the expected date $t+1$ return of agent j in his invested capital at date t. Log linearization of (10c) leads to

$$\gamma\hat{c}^j_t - \zeta(1-\gamma)\hat{\ell}^j_t = \mathrm{E}^j_t[-\hat{R}^j_{t+1} + \gamma\hat{c}^j_{t+1} - \zeta(1-\gamma)\hat{\ell}^j_{t+1}]. \tag{31b}$$

Subtract (31b) from (31a) to have

$$\hat{r}_t - \mathrm{E}^j_t[\hat{\pi}_{t+1}] = \mathrm{E}^j_t[\hat{R}_{t+1}]. \tag{32}$$

We now average (32) over the large number of N agents in the economy to write

$$\hat{r}_t - \bar{\mathrm{E}}_t[\hat{\pi}_{t+1}] = \frac{1}{N}\sum_{i=1}^{N}\mathrm{E}^j_t[\hat{R}^j_{t+1}]. \tag{33}$$

The crucial term is the sum of individual expected real rates of return on capital on the right side of Eqs. (33). We can rewrite the equation in the form

$$\hat{r}_t - \bar{\mathrm{E}}_t[\hat{\pi}_{t+1}] = \frac{1}{N}\sum_{j=1}^{N}[\bar{\mathrm{E}}[\hat{R}_{t+1}] + \{\mathrm{E}_t^j[\hat{R}_{t+1}^j] - \bar{\mathrm{E}}[\hat{R}_{t+1}]\}] \quad \text{where}$$

$$\hat{R}_{t+1} = \frac{1}{N}\sum_{j=1}^{N}\hat{R}_{t+1}^j$$

or

$$\hat{r}_t - \bar{\mathrm{E}}_t[\hat{\pi}_{t+1}] = \bar{\mathrm{E}}[\hat{R}_{t+1}] + \Delta_t \quad \text{where } \Delta_t = \frac{1}{N}\sum_{j=1}^{N}[\mathrm{E}_t^j[\hat{R}_{t+1}^j] - \bar{\mathrm{E}}[\hat{R}_{t+1}]]. \tag{34}$$

If beliefs are homogenous $\bar{\mathrm{E}} = \mathrm{E}^j$, all j and $\Delta_t = 0$. In this case we have the Fisher equation

$$\hat{r}_t - \bar{\mathrm{E}}_t[\hat{\pi}_{t+1}] = \bar{\mathrm{E}}[\hat{R}_{t+1}]. \tag{35}$$

It says *the real rate equals the nominal rate minus expected inflation* and this what we expect to find in the log linearized economy with homogenous beliefs. In an economy with full monetary dichotomy the policy rule has no real effect hence the real rate is invariant to policy; changes in monetary policy rules change only inflation and the right hand side of (35) *is invariant to policy*.

If beliefs are heterogenous, (34) says that $\Delta_t \neq 0$ implying that monetary policy alters the real rate via the difference between the expected rate of return of individual agents $\mathrm{E}_t^j[\hat{R}_{t+1}^j]$ and the market expected rate $\mathrm{E}_t[\hat{R}_{t+1}]$. The mean discrepancy Δ_t is the vehicle by which a monetary policy rule transforms diverse expectations into real effect of policy on the real rate. Even under sticky prices or other form of rigidity monetary policy affects the real rate by altering the expectations of agent who incorporate the monetary rule into their forecasting models. When agents have diverse beliefs a given monetary rule has a diverse effect on the expectations of agents hence causes a change in their employment, investment and consumption plans. The larger is the aggregate deviation of individual expectations from market expectation the stronger is the real effect of a monetary rule.

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programs used to compute it are available to the public on Mordecai Kurz's web page at http://www.stanford.edu/~mordecai.

Appendix A. Construction of the random variables $\eta^j_{t+1}(u^j_t)$

The variables $\eta^j_{t+1}(u^j_t)$ are our tools to enable agents to exhibit subjective beliefs with 'fat' tails reflecting over confidence. We define $\eta^j_{t+1}(u^j_t)$ by specifying its density, conditional on u^j_t:

$$p(\eta^j_{t+1}|u^j) = \begin{cases} \psi_1(u^j)\Phi(\eta^j_{t+1}) & \text{if } \eta^j_{t+1} \geqslant 0 \\ \psi_2(u)\Phi(\eta^j_{t+1}) & \text{if } \eta^j_{t+1} < 0 \end{cases} \tag{A.1}$$

where η^j_{t+1} and $\tilde{\rho}^{g^j}_{t+1}$ (in (19)) are independent and

$$\Phi(\eta) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\eta^2/2}. \tag{A.2}$$

To show the agent can hold such a belief and be rational, we use the Conditional Stability Theorem (see Kurz and Schneider, 1996). A sufficient condition which the theorem proposes requires that if $G(u^j)$ is the empirical density of u^j then we need to ensure that

$$\int_{-\infty}^{\infty} p(\eta^j/u^j)G(u^j)\,\mathrm{d}u^j = \Phi(\eta^j) \quad \text{a Normal Density of } \eta^j \tag{A.3}$$

Eq. (A.3) follows from (A.1) and by the two conditions:

$$\int_{-\infty}^{\infty} \psi_1(u^j)G(u^j)\,\mathrm{d}u^j = 1 \quad \text{and} \quad \int_{-\infty}^{\infty} \psi_2(u^j)G(u^j)\,\mathrm{d}u^j = 1.$$

To explain, since the empirical distribution of u^j is normal, averaging over u^j generates a random variable $\bar{\eta}^j_{t+1}$ such that $\bar{\eta}^j_{t+1} \sim \mathrm{N}(0,1)$. Averaged components of $\Psi_{t+1}(u^j_t)$ (i.e. $\lambda_\upsilon(\cdot)\bar{\eta}^j_{t+1} + \tilde{\rho}^\upsilon_{t+1}$) are then normally distributed with mean 0 and variance determined by the rationality conditions.

As for interpreting $\eta^j_{t+1}(u^j_t)$, the functions $\psi_i(u^j)$ $i = 1,2$ allow agents to construct different probabilities of η^j_{t+1} being positive or negative, conditional upon u^j. $\psi_i(u^j)$, $i = 1,2$ are monotone: $\psi_1(u^j)$ *is rising* with u^j towards 2 and $\psi_2(u^j)$ *is declining* with u^j towards 0. When they converge rapidly to their asymptotic values the densities take the form

(i) For large positive u^j (or g^j) the mean value of the density becomes positive since

$$p(\eta^j_{t+1}|u^j \text{ very large}) \approx \begin{cases} 2\Phi(\eta^j_{t+1}) & \text{for } \eta^j_{t+1} \geqslant 0 \\ 0 & \text{for } \eta^j_{t+1} < 0 \end{cases} \tag{A.4a}$$

(ii) For large negative u^j (or g^j) the mean value of the density becomes negative since

$$p(\eta^j_{t+1}|u^j \text{ very small}) \approx \begin{cases} 0 & \text{for } \eta^j_{t+1} \geqslant 0 \\ 2\Phi(\eta^j_{t+1}) & \text{for } \eta^j_{t+1} < 0 \end{cases}. \tag{A.4b}$$

As a practical approximation we have selected the function

$$\psi(u^j) = \frac{1}{1+\mathrm{e}^{bu^j(g^j)}}, \quad B = \int_{-\infty}^{\infty} \psi(u^j)G(u^j)\,\mathrm{d}u^j = \frac{1}{2}, \quad \psi_1(u) = 2\psi(u). \tag{A.4c}$$

Direct calculations reveal the three empirical moments of the random variable η^j under discussion

$$\mathrm{E}[\eta^j] = 0, \quad \mathrm{E}[(\eta^j)^2] = 1, \quad \mathrm{E}[\eta^j u^j] = \frac{4}{\sqrt{2\pi}}\,\mathrm{E}[\psi(u^j)u^j].$$

The parameter b is the *over confidence parameter* as it specifies the degree of fat tails in the conditional distribution of $\eta^j_{t+1}(u^j_t)$. In the discussion of the symmetric case below we shall use the notation $\sigma_u^2 = \mathrm{Var}(u)$ and $r_{\eta u} = \mathrm{E}\left[\frac{\eta_{t+1}(u)u}{\sigma_u}\right]$.

Appendix B. Statement of the rationality conditions

The rationality of belief principle requires that

$$\Psi_{t+1}(u^j_t) = \begin{pmatrix} \lambda^{\upsilon}_g \eta^j_{t+1}(u^j_t) + \tilde{\rho}^{\upsilon^j}_{t+1} \\ \lambda^{\varrho}_g \eta^j_{t+1}(u^j_t) + \tilde{\rho}^{\varrho^j}_{t+1} \\ \lambda^{z^1}_g \eta^j_{t+1}(u^j_t) + \tilde{\rho}^{z^{j1}}_{t+1} \\ \lambda^{z^2}_g \eta^j_{t+1}(u^j_t) + \tilde{\rho}^{z^{j2}}_{t+1} \end{pmatrix} \text{ has the same joint empirical distribution as}$$

$$\rho_{t+1} = \begin{pmatrix} \rho^{\upsilon}_{t+1} \\ \rho^{\varrho}_{t+1} \\ \rho^{z^1}_{t+1} \\ \rho^{z^2}_{t+1} \end{pmatrix} \tag{B.1}$$

when the sequence $\{u^j_t,\ t = 1, 2, \ldots\}$ is considered as part of the variability of the term on the left.

To clarify the mathematical development below keep in mind the consistency conditions between g^j and z^j. These conditions require that the *realizations* of the two are the same and hence they have the same marginal empirical distribution. Anonymity requires agents to ignore this fact and hence we assume it not be known to the agents. Technically speaking, the equalities $g^j_t = z^j_t$ for all t are treated as macroeconomic consistency condition (like market clearing conditions) but they do not hold in the agent's perception model who treats the z_t^j as macroeconomic variables (like prices) with their own perceived transition functions. Hence, in the agent's perception model there is nothing to require that the covariance between g^j and any state variable be the same as the covariance implied by the system (17) between z^j and that variable. Indeed, the presence of $\eta_{t+1}^j(u_t^j)$ in all equations of the perception model (21a) generates covariance between g^j and other state variables which is perceived by agent j but may not be present in (17). The idea is that any covariance between an agent's own state of belief and other variables in the economy *are strictly in the mind of the agent and no rationality conditions are imposed on them*. An important way to understand this is to note that under the condition of anonymity an agent sees no relationship between his own state of belief and the market vector of belief. Below we give a specific mathematical implication of anonymity.

We show first that the rationality condition (B.1) fully pins down the covariance matrix $\Omega_{\rho\rho}^j$ of the four dimensional vector $\tilde{\rho}_{t+1}^j$ in the perception models (21a)–(21b). We write $\Omega_{\rho\rho}^j = \Omega_{\rho\rho}$ all j since in this paper we consider only symmetric equilibria. It follows from the development here that the conditions can easily be generalized to non symmetric equilibria. To directly demonstrate why $\Omega_{\rho\rho}$ is pinned down by (B.1), use it to rewrite (21a) in the form

$$x_{t+1}^j = Ax_t + \lambda_g \eta_{t+1}^j(u_t^j) + \tilde{\rho}_{t+1}^j, \quad \lambda_g = (\lambda_g^{v}, \lambda_g^{\varrho}, \lambda_g^{z^1}, \lambda_g^{z^2}). \tag{B.2}$$

Now define $\sigma_\eta^2 = \mathrm{E}[(\eta_{t+1}^j(u_t^j))^2]$ and recall that V is the covariance matrix of x_t according to the empirical distribution (17). Computing the covariance matrix in (B.2) and equating the computed value to V leads to the equality $V = AVA' + \lambda_g(\lambda_g)'\sigma_\eta^2 + \Omega_{\rho\rho}$ which means that

$$\Omega_{\rho\rho} = V - AVA' - \lambda_g(\lambda_g)'\sigma_\eta^2. \tag{B.3}$$

Eq. (B.3) shows that given vector of parameters (b, λ_g) all magnitudes on the right of (B.3) are known and this pins down the covariance matrix $\Omega_{\rho\rho}$.

We now observe that from the point of view of the agent, his perception model includes the transition equation for g_{t+1}^j specified in (15). It follows that in the model of the agent we need to specify the full joint distribution of *five* variables: the four basic observables x_t together with the agent state of belief g_t^j. That is, the agent's perception model is specified by a 5×5 covariance matrix Ω of the innovations. We write this matrix Ω in the block form

$$\Omega = \begin{pmatrix} \Omega_{\rho\rho}, & \Omega_{xg} \\ \Omega_{xg}, & \sigma_g^2 \end{pmatrix},$$

where $\Omega_{xg} = [\mathrm{Cov}(\tilde{\rho}_{t+1}^{v}, \tilde{\rho}_{t+1}^{g^j}), \mathrm{Cov}(\tilde{\rho}_{t+1}^{\varrho}, \tilde{\rho}_{t+1}^{g^j}), \mathrm{Cov}(\tilde{\rho}_{t+1}^{z^1}, \tilde{\rho}_{t+1}^{g^j}), \mathrm{Cov}(\tilde{\rho}_{t+1}^{z^2}, \tilde{\rho}_{t+1}^{g^j})]$ is a 4×1 covariance vector of the innovations of the agent's belief and the innovations of the observables in the agent's perception model (see (21a)). In addition we now define the following:

$r^j = \mathrm{Cov}(x, g^j)$ − the unconditional covariance vector between g^j and the four observables x;

$a^j = (\lambda_v^{z^j}, \lambda_\varrho^{z^j}, 0, 0)$ − the vector of x parameters in the g^j Eq. (15).

By symmetry, both terms are the same for all j. To compute r we multiply (15) by the first four equations in (21a) and compute the four equations which define the unconditional covariance as

$$r = \lambda_z Ar + AVa + \lambda_z \lambda_g r_{\eta u} \sigma_u + \Omega_{xg}$$

hence the 4×1 covariance vector of the innovations of the agent's belief must satisfy

$$\Omega_{xg} = r - \lambda_z Ar - AVa + \lambda_z \lambda_g r_{\eta u} \sigma_u, \tag{B.4}$$

where

$$r_{\eta u} = \mathrm{E}\left[\frac{\eta_{t+1}(u)g}{\sigma_u}\right] = \mathrm{E}\left[\frac{\eta_{t+1}(u)u}{\sigma_u}\right]$$

(B.4) shows that Ω_{xg} is pinned down if we specify $r \equiv \mathrm{Cov}(x, g^j)$. However, our approach is to treat (B.4) as a system of four equations in the eight unknown (r, Ω_{xg}) so that for the moment we have specified only four restrictions (B.4). We now show that our theory provides four additional restrictions to determined (r, Ω_{xg}).

To explore the restrictions the theory imposes we proceed in two steps. We first utilize the definition of anonymity which requires the agent not to associate g_t^j with the stochastic properties of the corresponding market belief variable z_t^j. Anonymity has two simple implications which we exploit to deduce the needed restrictions:

Covariance implications of anonymity: *Anonymity* requires the idiosyncratic component of an agent's belief not to be correlated with market beliefs. It also implies that in a symmetric equilibrium the unconditional correlation between an agent's belief and the belief of 'others' is the same across agents. An agent's belief may, however, be correlated with the average 'market belief'. We translate this to require that

$$\Omega_{z^1 g^j} = \mathrm{Cov}(\tilde{\rho}_{t+1}^{z^1}, \tilde{\rho}_{t+1}^{g^j}) = 0, \tag{B.5a}$$

$$\Omega_{z^2 g^j} = \mathrm{Cov}(\tilde{\rho}_{t+1}^{z^2}, \tilde{\rho}_{t+1}^{g^j}) = 0, \tag{B.5b}$$

(B.5a)–(B.5b) restricts the vector r to satisfy $\Omega_{(xg)_3} = \Omega_{(xg)_4} = 0$.

To complete the determination of (r, Ω_{xg}) we need two more restriction and we now show that these are deduced from the *rationality condition requiring no serial correlation of* $\Psi_{t+1}(u_t^j)$. We first show that for $\Psi_{t+1}(u_t^j)$ to exhibit no serial correlation it is sufficient that it is uncorrelated with date t public information. To see why, recall that $\Psi_{t+1}(u_t^j) = x_{t+1} - Ax_t$ and assume that $\Psi_{t+1}(u_t^j)$ is uncorrelated with any observables up to date t. Hence

$$\begin{aligned}\mathrm{E}[\Psi_{t+1}(u_t^j)\Psi_t(u_{t-1}^j)] &= \mathrm{E}[\Psi_{t+1}(u_t^j)(x_t - Ax_{t-1})]\\ &= \mathrm{E}[\Psi_{t+1}(u_t^j)x_t] - \mathrm{E}[\Psi_{t+1}(u_t^j)Ax_{t-1}] = 0.\end{aligned}$$

To ensure that these conclusions hold we need to put restrictions on r which imply that u_t^j are not correlated with any x_{t-i}, for all $i \geqslant 0$. Recall first that by the definition of the filter u_t^j we have

$$u_t^j \equiv u(g_t^j) = g_t^j - r'V^{-1}x_t.$$

Hence, the requirement $\mathrm{Cov}(u_t^j, x_t) = 0$ is a simple implication of the filter since $r_t^j \equiv \mathrm{Cov}(x, g^j)$. To examine the requirement $\mathrm{Cov}(u_{t+1}^j, x_t) = 0$, recall that the agent's model (21a)–(21b) specifies

$$x_{t+1}^j = Ax_t + \lambda_g \eta_{t+1}^j(u_t) + \tilde{\rho}_{t+1}^j.$$

Hence we have

$$\begin{aligned}u_{t+1}^j \equiv u(g_{t+1}^j) &= g_{t+1}^j - r'V^{-1}x_{t+1}^j\\ &= \lambda_z g_t^j + a'x_t + \tilde{\rho}_{t+1}^{g^j} - r'V^{-1}(Ax_t + \lambda_g \eta_{t+1}^j(u_t) + \tilde{\rho}_{t+1}^j).\end{aligned}$$

Consequently, the condition $\mathrm{Cov}(u_{t+1}^j, x_t) \equiv E[u_{t+1}^j x_t'] = 0$ would be satisfied if

$$\lambda_z r' + a'V - r'V^{-1}AV = 0. \tag{B.6}$$

Although (B.6) is a system of four equations, we now show that (B.6) are the last two restrictions implied by the rationality of belief conditions. To see this fact note that since V is an invertible matrix, the equations (B.6) can be solved for the covariance vector r, implying

$$[A' - \lambda_z I]V^{-1}r' = a. \tag{B.7}$$

We study here only the symmetric case $\lambda_{z^1} = \lambda_{z^2} = \lambda_z$. In this case we have that the matrix A takes the following form

$$A' = \begin{pmatrix} \lambda_\upsilon, & 0, & \lambda_\upsilon^{z_1}, & \lambda_\upsilon^{z_2}\\ 0, & \lambda_\varrho, & \lambda_\varrho^{z_1}, & \lambda_\varrho^{z_2}\\ 0, & 0, & \lambda_z, & 0\\ 0, & 0, & 0, & \lambda_z \end{pmatrix}$$

hence $[A' - \lambda_z I]$ is singular with the last two rows being zero. This is compatible with the fact that $a = (\lambda_\upsilon^z, \lambda_\upsilon^z, 0, 0)$ hence, system (B.7) consists of only two restrictions.

We can conclude that (B.4) together with the conditions $\Omega_{(xg)_3} = \Omega_{(xg)_4} = 0$ and (B.7) completely determine (r, Ω_{xg}). Finally, when r is known, $\tilde{\sigma}_{g^j}^2$ is pinned down as follows. Since we know that $\sigma_{u^j}^2 = \mathrm{var}(g^j) - r'V^{-1}r$, we use the condition $\mathrm{var}(g^j) = \mathrm{var}(z^j)$ to compute

$$\tilde{\sigma}_{g^j}^2 = (1 - \lambda_z^2)\,\mathrm{var}(g^j) - a'Va - 2\lambda_z a'r.$$

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